

Number Theory

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Grades 9 - 12

Executive summary:

It is our intent to use the lessons in this unit to enrich our current curriculums, and not be used necessarily in any given order, but when when it comes up in our classroom. The activities we have chosen cover the following standards:

9.4.3.1 Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.

6.1.1.6 Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

6.1.1.5 Factor whole numbers; express a whole number as a product of prime factors with exponents.

8.2.2.4 Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

8.2.2.5 Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

3.1.2.1 Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.

3.1.2.2 Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.

5.1.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication and division of multidigit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.

3.1.1.2 Use place value to describe whole numbers between 1000 and 100,000 in terms of ten thousands, thousands, hundreds, tens and ones place

4.1.2.4 Read and write decimals with words and symbols; use place value to describe decimals in terms of thousands, hundreds, tens, ones, tenths, hundredths and thousandths.

Overview of Days:

Rabbit Problem: Students will find the fibonacci sequence

Jugs: Students will use relatively prime numbers to solve a problem

Contact: Students will use prime factorization and the GCF

Father of the Bride: Students will use prime factorization and the LCM

Candy Factory: Students will look at sequences and patterns

Spend \$1,000,000: Students will have to spend \$1,000,000

Cryptarithms: Students will explore how addition works with regrouping in a word puzzle

How Grand is your total?: Students will look at the operations of multiplication, division, radicals, factorial and use them to find the biggest number total

Guess My Number: Students will use knowledge of place value to and written clues to guess a number as well as create their own clues.

Four 4's: Students will use order of operations, and only 4's to create the numbers 1-50

Students will work with numerous situations to practice, explore, and understand many different examples of number theory. Students will have the opportunity to learn useful real world and problem solving strategies. The students will work as an individual, in small groups and as a class to discover the properties and relationships of numbers.

MCA Questions they should be able to answer after the unit:

4. What is the prime factorization of 630?

A. $2 \times 3 \times 5 \times 7$

B. $2 \times 3^2 \times 5 \times 7$

C. $2 \times 3^2 \times 35$

D. $2 \times 5 \times 7 \times 9$

16. An expression is shown.

$$4 + 3(6 + 10) \div 2$$

What is the value of the expression?

A. 16

B. 26

C. 28

D. 56

19. What is the greatest common factor of 48 and 64?

A. 2

B. 8

C. 16

D. 24

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Number Theory Post-Test

Lesson: Rabbit Problem

Benchmark: 9.4.3.1 Select and apply counting procedures, such as the multiplication and addition principles and tree diagrams, to determine the size of a sample space (the number of possible outcomes) and to calculate probabilities.

Launch: I have 2 bunnies that were given to me as a present. I thought I would let them go into the field that is behind my house. I am a little worried though about them getting into my garden. If a pair of baby rabbits are put into a field, how many pairs will there be: a) at the end of each month, and b) at the end of one year? Criteria: Rabbits are fully-grown at 1 month and have another pair of bunnies at 2 months. Each pair is comprised of 1 male and 1 female and no rabbits die or leave the field.

Explore: Students will work in groups of 2 to 3. Some questions to ask during work time: Can you see how the series is formed and how it continues? What pattern do you see? If students have arrived at a solution earlier than others ask if they can represent the numbers in an equation.

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers.

So what is the pattern? In the Fibonacci Sequence, each term is the sum of the two preceding terms. For instance, $1 + 1 = 2$, $1 + 2 = 3$, $2 + 3 = 5$, $3 + 5 = 8$. The first 12 numbers in the Fibonacci Sequence are 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 and 144. What is the Fibonacci sequence? How are the numbers related in the sequence? What were some of the strategies that you used to find the sequence.

Summarize: We found the fibonacci sequence today. The Fibonacci sequence is found by the sum of the two preceding numbers. The series 1, 1, 2, 3, 5, 8,...

Lesson: Jugs

Benchmark: 6.1.1.6 Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

Launch: In the movie Die Hard With a Vengeance the two main characters are given the following riddle: You have to defuse a bomb by placing exactly 4 gallons of water on a sensor. The problem is, you only have a 5 gallon jug and a 3 gallon jug on hand! How do they do it?

[Jugs Worksheet](#)

Explore: You may want to have the worksheet 1 sided so that you can hand out one at a time, the back of the worksheet does go into an answer a little bit. Students will work in groups of 2 to 3. Some questions to ask: What jug did you start with? Does it matter which jug you start with? How do you know that amount is the exact amount? If students found one way to do it ask if they can find another way to do it? Is there a reason for the the jugs being 3 & 5 gallons to start with? What if you changed the size of the jugs?

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers. Discuss as a class why the problem works in the first place while looking at #4 on the worksheet.

Summarize: Today we found a few different ways to make exactly 4 gallons from a 3 & 5 gallon jug. The only reason that this worked was that the 2 jugs you start with must be relatively prime to each other. Relatively Prime means the GCD of both numbers is 1.

Lesson: Contact

Benchmark: 6.1.1.5 Factor whole numbers; express a whole number as a product of prime factors with exponents.

Launch: In the movie contact it is hypothesized that a message from an alien life form would be in math, they receive a message in prime numbers. Using the worksheet find how many prime numbers there are between 0 and 100

[Contact Worksheet](#)

Explore: Students will be working in groups of 2-4. Walking around be sure to check on the students on what numbers they are crossing off with the sieve, and have students compare their list with other groups before moving onto the prime factorization. Questions to ask: does it matter the order in which you write your prime factorization? Does it matter what numbers your start with?

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers.

Compare a couple of different methods that students used to find the prime factorization of 48.

Share the numbers less than 50 that have only 2 prime factors, see if there are any missing and any that may need to be added

Prime numbers are important to us especially in internet security because there is no quick way to find any given large prime number and to break it apart into its prime factorization.

Summarize: Today we learned about prime numbers and how to write a number in its prime factorization. We used exponents to simplify our notation, and it doesn't matter the path you take to do the prime factorization you should end up in the same place.

Lesson: Father of the Bride

Benchmark: 6.1.1.6 Determine greatest common factors and least common multiples. Use common factors and common multiples to calculate with fractions and find equivalent fractions.

Launch: Why is the number of hot dogs in a package not the same as the number of buns in a package? If hotdogs are 10 per package and buns are 8 per package how many would you need to buy?

[Father of the Bride Worksheet](#)

Explore: In groups of 2-4 students will explore the given problem. Some questions to ask: Method did you use? Is there another way you could've done it? Are there pros and cons to different processes. What is the answer to the problem called?

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers.

Look for various methods that students used the tree method, a list, or if anyone did prime factorization. If no one did prime factorization...to the whole class pose the question, is there another way that we could do this using the method we discussed in our last activity? What are the pros and cons of the different ways to find the LCM?

Summarize: Today we learned finding the greatest common multiple. We used a tree, a list and prime factorization. Prime factorization is a shorter process for bigger numbers, and can be used to find both the LCM and the GCF.

Lesson: Candy Factory

Benchmark: 8.2.2.4 Represent arithmetic sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

8.2.2.5 Represent geometric sequences using equations, tables, graphs and verbal descriptions, and use them to solve problems.

Launch: We have looked recently at the fibonacci sequence which is a special sequence. We are now going to explore some other sequences. On the Candy Factory worksheet you will explore another sequence find a term further down in the sequence, look for patterns. See if you can find an equation to use to save you some time in finding your answers.

[Candy Factory Worksheet](#)

Explore: Students working in groups of 2-4 working on the candy factory worksheet. Some questions to ask: How did you come about getting your next number? How do you know that is the next number? What patterns do you see in your numbers? Is the pattern an addition/subtraction or multiplication/division pattern? Is there a way that we would write the sequence as a rule so that you can find any given number in the sequence?

Share: After making observations the teacher should select a few pieces of sample work. Ask those students to present how they arrived at their answers.

Compare a couple of different methods that students arrived at for equations to find $3b$ & $3c$ on the worksheet.

Discuss the strategies that helped students come up with the rules or some patterns they noticed.

After going through the equations students derived discuss the following:

Does it matter what type of equation you come up with? Are their advantages and disadvantages to the type of equation you use? How about finding the sum of a sequence what if you have a huge set a numbers and we don't want to add them all up? Are there any short cuts you could try?

Summarize: Today we looked at patterns in numbers and being able to derive some equations for those sets of numbers being able to find the sum of a sequence, and find a given number in a sequence.

Lesson: Spend \$1,000,000

[Spend \\$1,000,000 Worksheet](#)

Benchmark: 3.1.2.1 Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.

3.1.2.2 Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.

Launch: \$1,000,000 WOW! How amazing would that be to win \$1,000,000! I just received a flyer (link above) about a contest. This contest is open to everyone and begins today. The object of the contest is to spend exactly \$1,000,000 on as many items as possible. There are many things that should be purchased with the \$1,000,000. You will need somewhere new to live, something to drive, electronics, and the list goes on and on! How many things can you buy with the \$1,000,000 and your total needs to be exactly \$1,000,000. You will need to make a presentation to the Contest Committee with pictures, location of purchase, and prices of all your intended purchases. Don't forget to show all of your work with the \$1,000,000, accuracy will be judged as well. Good luck and happy spending!

Explore: Students will work with a partner on this project in an attempt to spend \$1,000,000. They will have internet access, catalogs, flyers, newspapers, etc available for them.

Share: After the groups have completed the project, four groups will share with each other and determine which project is the best of the four to advance on to the finals of the classroom. Each group of four will determine how they will decide the best on their four. The three finalist will present their project to the class, for the class to vote on the best one. What was the best way to keep track of how much you spend?

Summarize: We did a lot of real life investigation today into how much things cost and how quickly you could spend \$1,000,000. To win the contest you also had to be very accurate in your math skills of addition and subtraction.

Lesson: Cryptarithms

[Cryptarithms Website](#)

Benchmark: 3.1.2.1 Add and subtract multi-digit numbers, using efficient and generalizable procedures based on knowledge of place value, including standard algorithms.

3.1.2.2 Use addition and subtraction to solve real-world and mathematical problems involving whole numbers. Use various strategies, including the relationship between addition and subtraction, the use of technology, and the context of the problem to assess the reasonableness of results.

Launch: Today we are going to do something a little different. We are going work on some Cryptarithms puzzles. Cryptarithms puzzles are mathematical puzzles but the numbers are replaced with something else. Today the numbers are going to be replaced with letters. Now there are a couple of rules for Cryptarithm puzzles.

Rules

1. Each letter represents a unique number. It will be one and only one number.
2. Numbers must not start with a zero.
3. The solution is uniques, unless otherwise stated.

Explore: The students will work in groups of 3-4 to solve the puzzles. Put the puzzles up on the Board one at a time for the students to solve.

SO	TESS	SEND	FORTY	COCA
<u>+SO</u>	<u>+SEES</u>	<u>+MORE</u>	TEN	<u>+COLA</u>
TOO	ELLEN	MONEY	<u>+ TEN</u>	OASI S
			S I XTY	

Share: After making observations the teacher should select a sample work. Ask those students to present how they arrived at their answers. Where did you find it easiest to start the problem from the left or the right, or looking for a particular letter first? How did you know a zero would have to be in a place, or whether there would regrouping?

Summarize: Today we worked on solving Cryptarithms. It's fun to do something different! Students should use attributes of place value (look for regrouping, 0's, 1's, 9's, ...) to solve their cryptarithms.

Lesson: How Grand is My Total?

[How Grand is my Total from Bemidji State Website](#)

Benchmark: 5.1.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication and division of multidigit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.

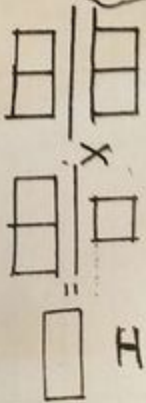
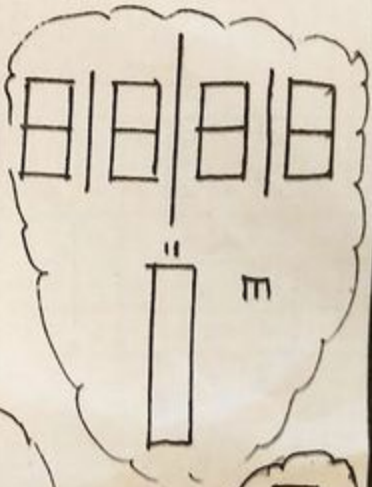
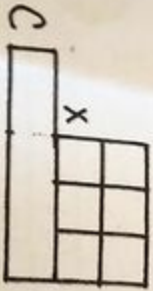
Launch: Today we are going to have a contest! We are going to work in groups of four to create the “Grandest” total. We will be using various math operations to get the highest number possible. Two restrictions. The first is that you can use the numbers 0-9 only a limited times on the sheet. The second is that you can not start a number with 0, such as 038. Have fun!

Explore: The students will work in groups of four to manipulate the numbers to create the grandest total. We have 2 versions of the worksheet. The one without radicals will be done first.

Share: As students complete their totals, they will write their total on the board. They can continue to work and increase their totals as new numbers are put up. What strategies did you use to increase your number? How did you start? Once you had a starting place then what did you do? Where did you put certain numbers?

Summarize: The last 2 days we looked at trying to achieve the greatest total that we could using multiplication, division, factorial, square roots. We found that you needed to keep your bigger numbers in the multiplication to go get the grandest. When dividing that you wanted to divided by the smallest number in order to achieve a larger number.

HOW GRAND IS YOUR TOTAL?



9	9	9	9	9	9	9	9	9	9
8	8	8	8	8	8	8	8	8	8
7	7	7	7	7	7	7	7	7	7
6	6	6	6	6	6	6	6	6	6
5	5	5	5	5	5	5	5	5	5
4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0



A	
B	
C	
D	
E	
F	
G	
H	
I	
GRAND TOTAL	

Lesson: Guess My Number

[Guess My Number Worksheets From CMP](#)

[Mystery Numbers Worksheet from Beacon Learning Center](#)

Benchmark: 3.1.1.2 Use place value to describe whole numbers between 1000 and 100,000 in terms of ten thousands, thousands, hundreds, tens and ones place

4.1.2.4 Read and write decimals with words and symbols; use place value to describe decimals in terms of thousands, hundreds, tens, ones, tenths, hundredths and thousandths.

Launch: If a rooster laid an egg on a slanted roof, which way would it roll? No way, roosters don't lay eggs! Haha, I know, cheesy! Does anyone else have a good appropriate riddle? Let a couple students tell riddles. Today we are going to do some math riddles and make our own.

Explore: Students will work in groups of four to solve the math riddles. They will check in with the master riddler (teacher) after each one to check their accuracy.

Share: After each group has completed their own math riddles, we will take turns with groups putting their riddle up on the board for the rest of the class to complete.

Summarize: Today we worked with math riddles, both solving them and creating them. We practiced place value, odds, evens, addition, subtraction, multiplication, multiples. Many found it helpful to write above a place value the numbers that could be there, and then go through the different options until another of the clues didn't hold up.

Lesson: 4 4's

Benchmark: 5.1.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication and division of multi-digit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.

Launch: If I have four cookies and I get four more. But then my dog jumps up and eats four of them. Four friends come over and I give them each a cookie. How many cookies do I have left? Ask a student to show on the board how they determined that I no longer have any cookies. Today we are going to be working with the number four, actually four fours to be exact. Please number your paper to 50. Your challenge today is to create equations that equal all the numbers from zero to fifty. You can use any mathematical operation but you can only use the number of 4 and it has to be used exactly 4 times.

Explore: The students will work in groups of fours to create equations.

Share: After making observations the teacher should select a sample work. Ask those students to present how they arrived at their answers. Which numbers were the easiest to create an equation for? Which ones were really difficult? Why?

Summarize: We used lots of math today. We manipulated the number 4 in so very many ways. You need to remember the order of operations PEMDAS. We found that there were several ways to find the same number using different operations of numbers.