STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Assignment 9
Chapter 4: 33, 36, 42, 44, 47, 56, 58.

4-33. a) E(X) = (-1+1)/2 = 0,

$$V(X) = \frac{(1-(-1))^2}{12} = 1/3, \text{ and } \sigma_x = 0.577$$

b) $P(-x < X < x) = \int_{-x}^{x} \tfrac{1}{2}\,dt = 0.5t\Big|_{-x}^{x} = 0.5(2x) = x$

Therefore, x should equal 0.90.

c) $F(x) = \begin{cases} 0, & x < -1 \\ 0.5x + 0.5, & -1 \le x < 1 \\ 1, & 1 \le x \end{cases}$

4-36. $E(X) = \dfrac{(1.5 + 2.2)}{2} = 1.85 \min$

$$V(X) = \frac{(2.2 - 1.5)^2}{12} = 0.0408 \min^2$$

b) $P(X < 2) = \int_{1.5}^{2} \frac{1}{(2.2-1.5)}dx = \int_{1.5}^{2} (1/0.7)dx = (1/0.7)x\Big|_{1.5}^{2} = (1/0.7)(0.5) = 0.7143$

c.) $F(X) = \int_{1.5}^{x} \frac{1}{(2.2-1.5)}dy = \int_{1.5}^{x} (1/0.7)dy = (1/0.7)y\,\Big|_{1.5}^{x}$ for 1.5 < x < 2.2. Therefore,

$$F(x) = \begin{cases} 0, & x < 1.5 \\ (1/0.7)x - 2.14, & 1.5 \le x < 2.2 \\ 1, & 2.2 \le x \end{cases}$$

4-42. a) $P(-1 < Z < 1) = P(Z < 1) - P(Z > 1)$

$$= 0.84134 - (1 - 0.84134)$$

$$= 0.68268$$

b) $P(-2 < Z < 2) = P(Z < 2) - [1 - P(Z < 2)]$

$$= 0.9545$$

c) $P(-3 < Z < 3) = P(Z < 3) - [1 - P(Z < 3)]$

$$= 0.9973$$

d) $P(Z > 3) = 1 - P(Z < 3)$

$$= 0.00135$$

e) $P(0 < Z < 1) = P(Z < 1) - P(Z < 0)$

$$= 0.84134 - 0.5 = 0.34134$$

4-44. a) Because of the symmetry of the normal distribution, the area in each tail of the distribution must equal 0.025. Therefore the value in Table III that corresponds to 0.975 is 1.96. Thus, $z = 1.96$.

b) Find the value in Table III corresponding to 0.995. $z = 2.58$.

c) Find the value in Table III corresponding to 0.84. $z = 1.0$

d) Find the value in Table III corresponding to 0.99865. $z = 3.0$.

4-47. a) $P(X < 11) = P\left(Z < \frac{11-5}{4}\right)$

$= P(Z < 1.5)$
$= 0.93319$

b) $P(X > 0) = P\left(Z > \frac{0-5}{4}\right)$

$= P(Z > -1.25)$
$= 1 - P(Z < -1.25)$
$= 0.89435$

c) $P(3 < X < 7) = P\left(\frac{3-5}{4} < Z < \frac{7-5}{4}\right)$

$= P(-0.5 < Z < 0.5)$
$= P(Z < 0.5) - P(Z < -0.5)$
$= 0.38292$

d) $P(-2 < X < 9) = P\left(\frac{-2-5}{4} < Z < \frac{9-5}{4}\right)$

$= P(-1.75 < Z < 1)$
$= P(Z < 1) - P(Z < -1.75)]$
$= 0.80128$

e) $P(2 < X < 8) = P\left(\frac{2-5}{4} < Z < \frac{8-5}{4}\right)$

$=P(-0.75 < Z < 0.75)$
$= P(Z < 0.75) - P(Z < -0.75)$
$= 0.54674$

4-56. a) $P(X > 0.5) = P\left(Z > \frac{0.5 - 0.4}{0.05}\right)$

$= P(Z > 2)$
$= 1 - 0.97725$
$= 0.02275$

b) $P(0.4 < X < 0.5) = P\left(\frac{0.4 - 0.4}{0.05} < Z < \frac{0.5 - 0.4}{0.05}\right)$

$= P(0 < Z < 2)$
$= P(Z < 2) - P(Z < 0)$
$= 0.47725$

c) $P(X > x) = 0.90$, then $P\left(Z > \frac{x - 0.4}{0.05}\right) = 0.90$.

Therefore, $\frac{x-0.4}{0.05} = -1.28$ and $x = 0.336$.

4-58. Let X denote the height.
$X \sim N(64, 2^2)$

(a) $P(58 < X < 70) = \Phi(\frac{70-64}{2}) - \Phi(\frac{58-64}{2}) = \Phi(3) - \Phi(-3) = 0.9973$

(b) $\Phi^{-1}(0.25) \times 2 + 64 = 62.6510$

$\Phi^{-1}(0.75) \times 2 + 64 = 65.3490$

(c) $\Phi^{-1}(0.05) \times 2 + 64 = 60.7103$

$\Phi^{-1}(0.95) \times 2 + 64 = 67.2897$

(d) $[1 - \Phi(\frac{68-64}{2})]^5 = [1 - \Phi(2)]^5 = 6.0942 \times 10^{-9}$