STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Assignment 8
Chapter 4: 4, 6, 9, 12, 16, 20, 26, 28.

4-4 a) $P(1 < X) = \int_4^{\infty} e^{-(x-4)}dx = -e^{-(x-4)}\Big|_4^{\infty} = 1$, because $f_X(x) = 0$ for x < 4. This can also be obtained from the fact that $f_X(x)$ is a probability density function for 4 < x.

b) $P(2 \le X \le 5) = \int_4^{5} e^{-(x-4)}dx = -e^{-(x-4)}\Big|_4^{5} = 1 - e^{-1} = 0.6321$

c) $P(5 < X) = 1 - P(X \le 5)$. From part b., $P(X \le 5) = 0.6321$. Therefore, $P(5 < X) = 0.3679$.

d) $P(8 < X < 12) = \int_8^{12} e^{-(x-4)}dx = -e^{-(x-4)}\Big|_8^{12} = e^{-4} - e^{-8} = 0.0180$

e) $P(X < x) = \int_4^{x} e^{-(x-4)}dx = -e^{-(x-4)}\Big|_4^{x} = 1 - e^{-(x-4)} = 0.90$.

Then, x = 4 − ln(0.10) = 6.303

4-6. a) $P(X > 3000) = \int_{3000}^{\infty} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{3000}^{\infty} = e^{-3} = 0.05$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233$

c) $P(X < 1000) = \int_{0}^{1000} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{0}^{1000} = 1 - e^{-1} = 0.6321$

d) $P(X < x) = \int_{0}^{x} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{0}^{x} = 1 - e^{-x/1000} = 0.10$.

Then, $e^{-x/1000} = 0.9$, and x = −1000 ln 0.9 = 105.36.

4-9 a) P(X < 2.25 or X > 2.75) = P(X < 2.25) + P(X > 2.75) because the two events are mutually exclusive. Then, P(X < 2.25) = 0 and

$$P(X > 2.75) = \int_{2.75}^{2.8} 2dx = 2(0.05) = 0.10\,.$$

b) If the probability density function is centered at 2.55 meters, then $f_X(x) = 2$ for $2.3 < x < 2.8$ and all rods will meet specifications.

4-12. a) $P(X < 1.8) = P(X \le 1.8) = F_X(1.8)$ because X is a continuous random variable. Then,

$$F_X(1.8) = 0.25(1.8) + 0.5 = 0.95$$

b) $P(X > -1.5) = 1 - P(X \le -1.5) = 1 - .125 = 0.875$

c) P(X < -2) = 0

d) $P(-1 < X < 1) = P(-1 < X \le 1) = F_X(1) - F_X(-1) = .75 - .25 = 0.50$

4-16. Now, $f(x) = x/8$ for 3 < x < 5 and $F_X(x) = \int_3^x \frac{x}{8}dx = \left.\frac{x^2}{16}\right|_3^x = \frac{x^2 - 9}{16}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, x < 3 \\ \frac{x^2 - 9}{16}, 3 \le x < 5 \\ 1, x \ge 5 \end{cases}$

4-20. $f(x) = 2e^{-2x},\ x > 0$

4-26. $$E(X) = \int_3^5 x\frac{x}{8}dx = \left.\frac{x^3}{24}\right|_3^5 = \frac{5^3 - 3^3}{24} = 4.083$$

$$V(X) = \int_3^5 (x - 4.083)^2 \frac{x}{8}dx = \int_3^5 \left(\frac{x^3}{8} - \frac{8.166x^2}{8} + \frac{16.6709x}{8}\right)dx$$

$$= \frac{1}{8}\left.\left(\frac{x^4}{4} - \frac{8.166x^3}{3} + \frac{16.6709x^2}{2}\right)\right|_3^5 = 0.3264$$

4-28. a)

$$E(X) = \int_{1200}^{1210} x0.1dx = 0.05x^2\Big|_{1200}^{1210} = 1205$$

$$V(X) = \int_{1200}^{1210} (x-1205)^2 0.1dx = 0.1\frac{(x-1205)^3}{3}\Bigg|_{1200}^{1210} = 8.333$$

$$\textit{Therefore,} \quad \sigma_x = \sqrt{V(X)} = 2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1195 < X < 1205) = P(1200 < X < 1205) = \int_{1200}^{1205} 0.1dx = 0.1x\Big|_{1200}^{1205} = 0.5$$