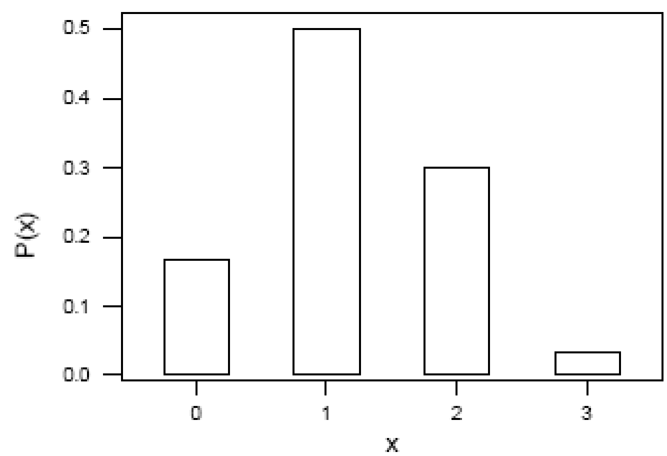
STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Assignment 7
Chapter 3: 99, 101, 104, 109, 111, 115.

3-99. N=10, n=3 and K=4



3-101. Let X denote the number of men who carry the marker on the male chromosome for an increased risk for high blood pressure. N=800, K=240 n=10

a) n=10

$$P(X=1)=\frac{\binom{240}{1}\binom{560}{9}}{\binom{800}{10}}=\frac{\left(\frac{240!}{1!239!}\right)\left(\frac{560!}{9!551!}\right)}{\frac{800!}{10!790!}}=0.1201$$

b) n=10

$$P(X>1)=1-P(X\le 1)=1-[P(X=0)+P(X=1)]$$

$$P(X=0)=\frac{\binom{240}{0}\binom{560}{10}}{\binom{800}{10}}=\frac{\left(\frac{240!}{0!240!}\right)\left(\frac{560!}{10!550!}\right)}{\frac{800!}{10!790!}}=0.0276$$

$$P(X>1)=1-P(X\le 1)=1-[0.0276+0.1201]=0.8523$$

3-104. Let X denote the count of the numbers in the state's sample that match those in the player's sample. Then, X has a hypergeometric distribution with N = 40, n = 6, and K = 6.

a) $P(X=6) = \dfrac{\binom{6}{6}\binom{34}{0}}{\binom{40}{6}} = \left(\dfrac{40!}{6!34!}\right)^{-1} = 2.61 \times 10^{-7}$

b) $P(X=5) = \dfrac{\binom{6}{5}\binom{34}{1}}{\binom{40}{6}} = \dfrac{6 \times 34}{\binom{40}{6}} = 5.31 \times 10^{-5}$

c) $P(X=4) = \dfrac{\binom{6}{4}\binom{34}{2}}{\binom{40}{6}} = 0.00219$

d) Let Y denote the number of weeks needed to match all six numbers. Then, Y has a geometric distribution with p =

$\dfrac{1}{3{,}838{,}380}$ and E(Y) = 1/p = 3,838,380 weeks. This is more than 738 centuries!

3-109. $P(X=0) = e^{-\lambda} = 0.05$. Therefore, $\lambda = -\ln(0.05) = 2.996$.
Consequently, $E(X) = V(X) = 2.996$.

3-111. $\lambda=1$, Poisson distribution. $f(x) = e^{-\lambda}\lambda^x/x!$
(a) $P(X \geq 2) = 0.264$
(b) In order that $P(X \geq 1) = 1 - P(X=0) = 1 - e^{-\lambda}$ exceed 0.95, we need $\lambda=3$.
Therefore 3*16=48 cubic light years of space must be studied.

3-115.a) $E(X) = \lambda = 0.2$ errors per test area

b) $P(X \leq 2) = e^{-0.2} + \dfrac{e^{-0.2}\,0.2}{1!} + \dfrac{e^{-0.2}(0.2)^2}{2!} = 0.9989$

99.89% of test areas