STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Assignment 5
Chapter 3: 15, 17, 23, 27, 29, 33, 35, 41, 43, 50.

3-15. All probabilities are greater than or equal to zero and sum to one.

a) $P(X \le 2) = 1/8 + 2/8 + 2/8 + 2/8 + 1/8 = 1$
b) $P(X > -2) = 2/8 + 2/8 + 2/8 + 1/8 = 7/8$
c) $P(-1 \le X \le 1) = 2/8 + 2/8 + 2/8 = 6/8 = 3/4$
d) $P(X \le -1 \text{ or } X=2) = 1/8 + 2/8 + 1/8 = 4/8 = 1/2$

3-17. Probabilities are nonnegative and sum to one.
a) $P(X = 4) = 9/25$
b) $P(X \le 1) = 1/25 + 3/25 = 4/25$
c) $P(2 \le X < 4) = 5/25 + 7/25 = 12/25$
d) $P(X > -10) = 1$

3-23. $P(X = 50 \text{ million}) = 0.5$, $P(X = 25 \text{ million}) = 0.3$, $P(X = 10 \text{ million}) = 0.2$

3-27. X = number of components that meet specifications
$P(X=0) = (0.05)(0.02)(0.01) = 0.00001$
$P(X=1) = (0.95)(0.02)(0.01) + (0.05)(0.98)(0.01)+(0.05)(0.02)(0.99) = 0.00167$
$P(X=2) = (0.95)(0.98)(0.01) + (0.95)(0.02)(0.99) + (0.05)(0.98)(0.99) = 0.07663$
$P(X=3) = (0.95)(0.98)(0.99) = 0.92169$

3-29.

$$F(x) = \begin{cases} 0, & x < -2 \\ 1/8 & -2 \le x < -1 \\ 3/8 & -1 \le x < 0 \\ 5/8 & 0 \le x < 1 \\ 7/8 & 1 \le x < 2 \\ 1 & 2 \le x \end{cases} \text{ where } \begin{aligned} f_X(-2) &= 1/8 \\ f_X(-1) &= 2/8 \\ f_X(0) &= 2/8 \\ f_X(1) &= 2/8 \\ f_X(2) &= 1/8 \end{aligned}$$

a) $P(X \le 1.25) = 7/8$
b) $P(X \le 2.2) = 1$
c) $P(-1.1 < X \le 1) = 7/8 - 1/8 = 3/4$
d) $P(X > 0) = 1 - P(X \le 0) = 1 - 5/8 = 3/8$

3-33.

$$F(x) = \begin{cases} 0, & x < 10 \\ 0.2, & 10 \le x < 25 \\ 0.5, & 25 \le x < 50 \\ 1, & 50 \le x \end{cases}$$

where P(X = 50 million) = 0.5, P(X = 25 million) = 0.3, P(X = 10 million) = 0.2

3-35. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero;
   pmf: $f(1) = 0.5, f(3) = 0.5$
   a) $P(X \le 3) = 1$
   b) $P(X \le 2) = 0.5$
   c) $P(1 \le X \le 2) = P(X=1) = 0.5$
   d) $P(X>2) = 1 - P(X \le 2) = 0.5$

3-41. Determine E(X) and V(X) for random variable in exercise 3-15
$$\mu = E(X) = -2f(-2) - 1f(-1) + 0f(0) + 1f(1) + 2f(2)$$
$$= -2(1/8) - 1(2/8) + 0(2/8) + 1(2/8) + 2(1/8) = 0$$
$$V(X) = -2^2 f(-2) - 1^2 f(-1) + 0^2 f(0) + 1^2 f(1) + 2^2 f(2) - \mu^2$$
$$= 4(1/8) + 1(2/8) + 0(2/8) + 1(2/8) + 4(1/8) - 0^2 = 1.5$$

3-43. Mean and variance for exercise 3-17
$$\mu = E(X) = 0f(0) + 1f(1) + 2f(2) + 3f(3) + 4f(4)$$
$$= 0(0.04) + 1(0.12) + 2(0.2) + 3(0.28) + 4(0.36) = 2.8$$
$$V(X) = 0^2 f(0) + 1^2 f(1) + 2^2 f(2) + 3^2 f(3) + 4^2 f(4) - \mu^2$$
$$= 0(0.04) + 1(0.12) + 4(0.2) + 9(0.28) + 16(0.36) - 2.8^2 = 1.36$$

3-50. μ=E(X)=350*0.06+450*0.1+550*0.47+650*0.37=565

$$V(X) = \sum_{i-1}^{4} f(x_i)(x - \mu)^2 = 6875$$
$$\sigma = \sqrt{V(X)} = 82.92$$