STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Solutions to Chapter 2: 89, 90, 91, 93, 95, 101, 103, 105, 107, 109, 114.

2-89. a) $P(A \cap B) = P(A|B)P(B) = (0.4)(0.5) = 0.20$

b) $P(A' \cap B) = P(A'|B)P(B) = (0.6)(0.5) = 0.30$

2-90.

$$
\begin{aligned}
P(A) &= P(A \cap B) + P(A \cap B') \\
&= P(A|B)P(B) + P(A|B')P(B') \\
&= (0.2)(0.8) + (0.3)(0.2) \\
&= 0.16 + 0.06 = 0.22
\end{aligned}
$$

2-91. Let F denote the event that a connector fails.
Let W denote the event that a connector is wet.

$$
\begin{aligned}
P(F) &= P(F|W)P(W) + P(F|W')P(W') \\
&= (0.05)(0.10) + (0.01)(0.90) = 0.014
\end{aligned}
$$

2-93. Let R denote the event that a product exhibits surface roughness. Let N,A, and W denote the events that the blades are new, average, and worn, respectively. Then,
$P(R)= P(R|N)P(N) + P(R|A)P(A) + P(R|W)P(W)$
$= (0.01)(0.25) + (0.03) (0.60) + (0.05)(0.15)$
$= 0.028$

2-95.a) $(0.88)(0.27) = 0.2376$
b) $(0.12)(0.13+0.52) = 0.0.078$

2-101. $P(A') = 1 - P(A) = 0.7$ and $P(A'|B) = 1 - P(A|B) = 0.7$
Therefore, A' and B are independent events.

2-103. a) $P(B|A) = 4/499$ and

$$P(B) = P(B|A)P(A) + P(B|A')P(A') = (4/499)(5/500) + (5/499)(495/500) = 5/500$$

Therefore, A and B are <u>not</u> independent.

b) A and B are independent.

2-105. a) $P(A \cap B) = 22/100$, $P(A) = 30/100$, $P(B) = 77/100$, Then $P(A \cap B) \neq P(A)P(B)$, therefore, A and B are not independent.

b) $P(B|A) = P(A \cap B)/P(A) = (22/100)/(30/100) = 0.733$

2-107. It is useful to work one of these exercises with care to illustrate the laws of probability. Let H_i denote the event that the ith sample contains high levels of contamination.

a) $P(H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5') = P(H_1')P(H_2')P(H_3')P(H_4')P(H_5')$

by independence. Also, $P(H_i') = 0.9$. Therefore, the answer is $0.9^5 = 0.59$

b) $A_1 = (H_1 \cap H_2' \cap H_3' \cap H_4' \cap H_5')$

$A_2 = (H_1' \cap H_2 \cap H_3' \cap H_4' \cap H_5')$

$A_3 = (H_1' \cap H_2' \cap H_3 \cap H_4' \cap H_5')$

$A_4 = (H_1' \cap H_2' \cap H_3' \cap H_4 \cap H_5')$

$A_5 = (H_1' \cap H_2' \cap H_3' \cap H_4' \cap H_5)$

The requested probability is the probability of the union $A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$ and these events are mutually exclusive. Also, by independence $P(A_i) = 0.9^4(0.1) = 0.0656$. Therefore, the answer is $5(0.0656) = 0.328$.

c) Let B denote the event that no sample contains high levels of contamination. The requested probability is $P(B') = 1 - P(B)$. From part (a), $P(B') = 1 - 0.59 = 0.41$.

2-109. (a) $3(0.2^4) = 0.0048$

(b) $3(4 * 0.2^3 * 0.8) = 0.0768$

2-114. Let A denote the upper devices function. Let B denote the lower devices function.

$P(A) = (0.9)(0.8)(0.7) = 0.504$

$P(B) = (0.95)(0.95)(0.95) = 0.8574$

$P(A \cap B) = (0.504)(0.8574) = 0.4321$

Therefore, the probability that the circuit operates $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.9293$