MODELS FOR INITIAL DECIMAL IDEAS

Put a twist on the familiar 10 × 10 grid model to build your students' understanding of effective decimal models.

By Kathleen A. Cramer, Debra S. Monson,
Terry Wyberg, Seth Leavitt, and
Stephanie B. Whitney

Appropriate concrete and pictorial models allow students to construct meaning for rational numbers and operations with the numbers. To develop deep understanding of rational number, sixth through eighth graders must experience a variety of models (NCTM 2000).

Since 1979, personnel from the Rational Number Project (RNP), a cooperative research and development project funded by the National Science Foundation, have been investigating children's learning of fractions, ratios, decimals, and proportionality. In their latest curriculum development project, this RNP group used a familiar 10 × 10 grid to build meaning for decimals and improve student understanding of addition and subtraction with decimals.

The model

For this (and a former) curriculum development project, the group used the Lesh Translation model (see **fig. 1**), which identifies five modes of representation: (1) real-world situations, (2) manipulatives, (3) pictures, (4) spoken symbols, and (5) written symbols (Cramer 2003). Conceptual understanding depends on students' experiences with representing mathematical ideas in each of these modes and translating within and between modes. A *translation* is the reinterpretation of a concept between different representations or within the same representation. The arrows between the ovals in **figure 1** indicate that students make translations between different representations. For example, when students model the decimal 0.34 using a 10 × 10 grid, they are translating from written symbols to the pictorial mode. The arrows within each oval indicate that students make translations within the same representation. For instance, when students model decimals using a 10 × 10 grid and show the same decimal using a number line, they are translating within the pictorial mode.

The RNP curriculum offers many opportunities for students to represent decimals in a variety of ways and to make connections within and between representations. After students learned to represent and name

FIGURE 1

The Lesh translation model shows that students make translations between different representations.

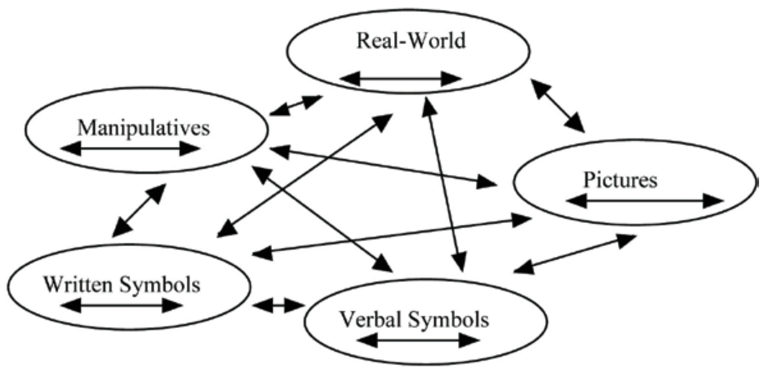


TABLE 1

Six of twenty-four lessons in the scope and sequence dealt with decimals.

Lesson	Lesson Goal
9	Students create a model for decimals using a 10 × 10 grid to show tenths and hundredths. They record amounts in words, fractions, symbols, and decimals.
10	Students develop an understanding of thousandths and begin to look at equivalence among tenths, hundredths, and thousandths. Students develop decimal order strategies by identifying the larger of two decimals, by sorting sets of decimals, and by finding a decimal between two decimals.
11	Students estimate sums and differences using mental images of a 10 × 10 grid. Students develop strategies for adding and subtracting decimals using a decimal +/– board. Students find actual answers to decimal addition and subtraction using mental math.
12	Students review ordering and equivalence and practice adding and subtracting decimals using a decimal +/– board, all within problem-solving contexts.
13	Students use a meter stick and number lines as models for decimals by connecting these new models to the 10 × 10 grid model.
14	Students model decimal addition and subtraction problems using a number line, a decimal +/– board, and symbols.

decimals using a 10 × 10 grid, they connected (translated) that representation to modeling decimals, first using a meter stick and then a number line. Using symbols, they recorded their actions on each model; using verbal and written language, they explained differences and similarities among the different representations. When adding and subtracting decimals, students solved problems embedded in measurement contexts. For example, when given a table with data on plant growth, students solved addition and subtraction story problems using one of the models. Then they used symbols on the model to record their actions. This process allows students to connect real-world items to concrete, pictorial representations and translate the pictures to symbols. The constant movement and intellectual activity in the Lesh translation model reflect a dynamic view of instruction and concept development.

Although all representations are important, this article shares insights as to which concrete and pictorial models have been effective in developing initial *decimal* ideas, which include understanding what decimals mean in terms of fractions and place value, constructing order strategies to judge the relative size of decimals, understanding decimal equivalence, and developing meaningful strategies for adding and subtracting decimals (as opposed to rote memorization of a procedure).

Initial decimal ideas

Early work by the RNP group showed how important mental images—that are based on concrete models—are in supporting students' meaningful work with fractions (Cramer, Post, and delMas 2002). This work indicates that fraction circles are uniquely effective in creating strong mental images for students, who use the mental images to make sense of order, equivalence, and fraction operations.

The RNP group wanted to find which models for decimals were effective in building strong mental images that would in turn support student work with order, equivalence, and operations with decimals. They tested two groups of sixth graders with a 10 × 10 grid and a number line. The impact of the 10 × 10 grid (and its adaptation) on students' understanding of initial decimal ideas is described below.

The decimal lessons are part of a new curriculum module that was developed and revised within two teaching experiments. This new module is a companion module to the RNP Fractions for the Middles Grades Level 1 materials also developed with NSF funding (Cramer, Behr, Post, and Lesh 1997). Briefly, the research team followed this sequence as they developed the module:

- **Teach** the first draft of the lessons to a class of sixth graders in an urban classroom during a six-week period.
- **Collect** data from classroom observations, class work, written assessments, and student interviews.
- **Revise** the curriculum module after careful analyses of students' thinking; re-teach it with another class of sixth graders for six weeks in the same urban district.
- **Revise** the module again on the basis of the second teaching experiment. Then eight sixth-grade teachers from the same district teach the lessons to their classes.
- **Revise** the curriculum module again after considering feedback from the classroom teachers.

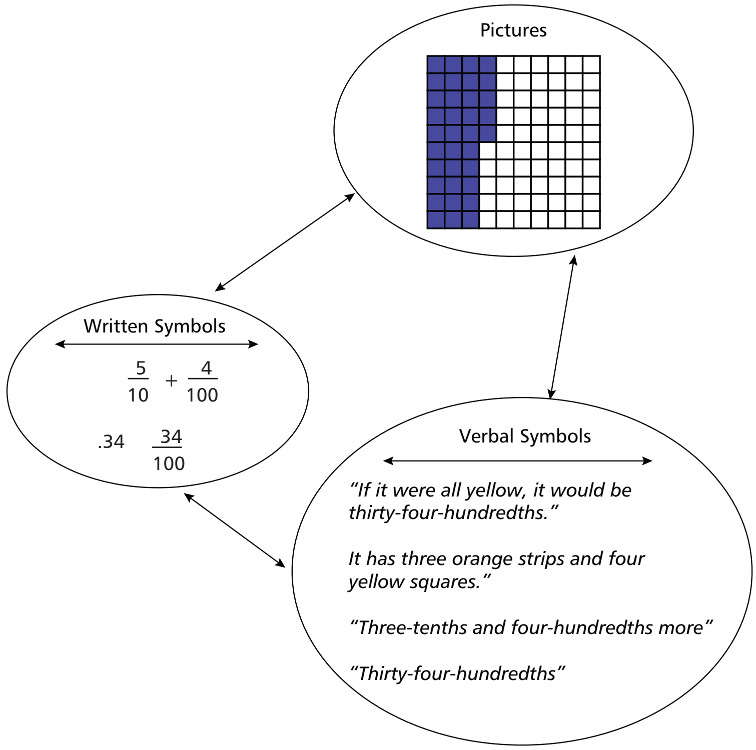
Six of the twenty-four lessons dealt with decimals and were covered in six or seven class periods. **Table 1** shows the scope and sequence for the decimal lessons.

The grid

The first model used to develop meaning for decimals was the familiar 10 × 10 grid. Students constructed a 10 × 10 grid in the first lesson and used orange and yellow crayons to color decimal amounts. To facilitate classroom discussion, the classroom teacher displayed a large poster of this model. In the first lesson, students built connections among various ways of describing different shaded amounts, starting with fractions. For example, students first showed 3 of 10 equal parts of a square partitioned into 10 equal parts. Then students described that amount in a variety of ways: by verbally calling it *three-tenths*, by writing the amount as the fraction 3/10, and by representing it as three orange strips.

After partitioning the same square into 100 equal parts to form a 10 × 10 grid, students shaded 2 of 10 equal parts plus 6 of 100 equal parts. Students' descriptions of this amount included writing 2/10 + 6/100 and 26/100; verbalizing *twenty-six-hundredths*, and representing the amount as two orange strips and six yellow squares or as twenty-six yellow squares. [*Editor's note:* For readability, all large fractions, including those that are spoken, are shown as mixed representations. That is, for the remainder of the article, fractions such as *twenty-six-hundredths* are written as *26 hundredths*, except in the figures, where spoken numbers are completely spelled out.]

FIGURE 2 The shaded areas of a 10 × 10 grid can be described in a variety of ways.



Language is important, and team members encouraged students to describe different examples for decimals in a variety of ways before they introduced decimal notation as a new way of recording what students already understood about fraction notation. For example, students showed 34 hundredths on their grids by using three orange strips and four yellow squares. They explained why it could be written as 3/10 and 4/100 more. They were asked to describe the same amount if it was shown as thirty-four yellow squares on the 10 × 10 grid. When team members introduced the symbol 0.34 for this amount, they encouraged students to make sense of the symbol by comparing it to the way they had expressed 34 hundredths as 3/10 + 4/100 and the representation with thirty-four yellow strips. Following work with tenths and hundredths, students extended the grid to show thousandths.

Students made connections among the 10 × 10 grid, language for describing amounts in different ways, fraction symbols, and decimal notation (see **fig. 2**). This emphasis on connections strengthens students' understanding of the decimal symbol and the amount it represents.

As a cautionary note, students initially struggled to keep track of whether the decimal is read as *tenths*, *hundredths*, or *thousandths*. The issue is muddied when students name a decimal such as 0.23 as *point twenty-three* instead of *23 hundredths*. We guided students away from using the former language.

The following example shows how one student was able to move between the correct language and the grid to overcome his initial misunderstanding of decimal equivalence. An interviewer asked the student to name 0.11 and 0.110 and determine either the larger number or that the numbers are equal. The interviewer's questions led the student back to the 10 × 10 model as a way to overcome his initial error.

S: Zero and 11 hundredths and then zero and 110 thousandths. And so this one [*pointing to 0.11*] is kind of like the less digits or numbers there are, the one with less is going to be bigger 'cause you are going to be dividing it into less pieces.

I: OK, so which one is going to be bigger?

S: Eleven-tenths … 11 hundredths. Zero and 11 hundredths.

I: And this would be … [*pointing to 0.110*]?

S: Zero and 110 thousandths.

I: Now, if you were going to shade that [*pointing to 0.11*], what would you shade?

S: [I would shade] 11 hundredths: one orange and one yellow.

I: How about that one [*pointing to 0.110*]?

S: Since you shade in 110 thousandths, you get one hundred in one square. You shade in ten. Um, I would do one.

I: One what?

S: One square.

I: This [*pointing to 0.110*] means one square?

S: Well, one and a little bit more.

I: A yellow square?

S: A yellow square because, no, because that wouldn't … well, no, I wouldn't. One-hundredth and then one-tenth and then zero thousandths.

I: So, what would you shade in?

S: I would shade in … there would be eleven 'cause maybe they are the same. 'Cause I would shade in one orange and then one yellow, and that's what I would do for that, so maybe they are the same.

I: So, are they the same?

S: Yeah, I think so.

Although the student was initially able to name the decimals using correct language, it was not until he made the connection between the decimal language and the model that he understood that the two numbers are equivalent. While students are still developing meaning for decimal symbols, the 10 × 10 grid is a reliable image to fall back on.

The use of color

The team found that using specific colors for shading tenths and hundredths was important. Previous work with fraction circles showed that students' mental images were strongly connected to the color of the fraction pieces. Team members paid attention to connecting specific colors to tenths and to hundredths when introducing the 10 × 10 grid for decimals to enrich whatever mental representations students created from this model.

To introduce students to thousandths, team members asked students to reflect on how they initially created their 10 × 10 grid. They had first partitioned the unit square into 10 equal parts. Then they had partitioned each tenth into 10 equal parts to divide the unit square into 100

FIGURE 3

Student work on thousandths involved ordering six decimal numbers.

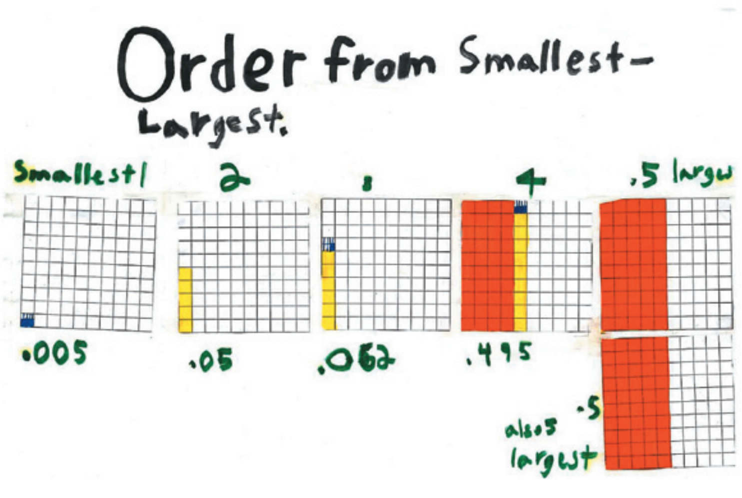
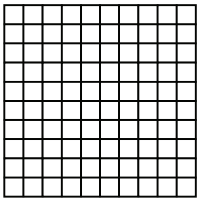


FIGURE 4

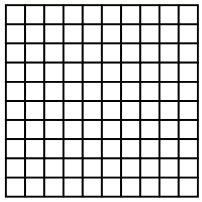
Sample problems show how to use a 10 × 10 grid to name decimals and to explore decimal order and equivalence.

On the grid, show and name the following in several ways:

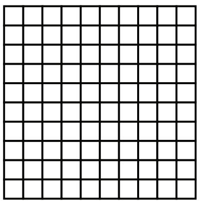


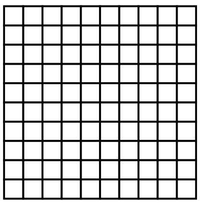
nine-tenths and eight-hundredths

Use the grid below to find three numbers between 0.07 and 0.08. Explain how you know the numbers are between these two values.

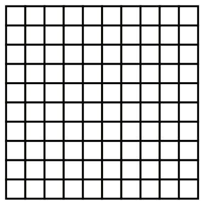


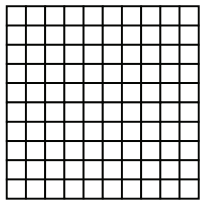
Circle the smaller of these two numbers:
.025 0.03
Explain how to use the grids below to support your answer.





After you determine whether the following expression is true or false, use the grids to support your answer: .880 = .8





Imagine each decimal below on a 10 × 10 grid. Describe each decimal. Which number is the smallest? How do you know?

0.625
0.25
0.675
0.8

Name three decimals that are less than 1 but greater than $\frac{1}{2}$. Describe what they would look like on a 10 × 10 grid. How do you know that they are greater than $\frac{1}{2}$?

equal parts. The team posed a question: "How can you partition the 10 × 10 grid to show thousandths?"

Students worked on this task in groups. Surprisingly, it was not obvious to students that they needed to divide each hundredths square into 10 equal parts. For example, one group tried partitioning each hundredths square into 4 equal parts. They divided a square into 4 equal parts and then mentally counted to find that their approach would lead to only 400 parts. They then divided a hundredth square into 8 equal parts. In the end, they realized that they needed to divide each hundredths square into 10 equal parts to partition the whole unit square into thousandths. Students were asked to use 10 × 10 grids to order 0.005, 0.50, 0.05, 0.495,0.062, and 0.5 (see **fig. 3**). A variety of activities that use the 10 × 10 grid can help students learn to name decimals and to build order and equivalence ideas for decimals (see **fig. 4**).

The model's effectiveness

Was the 10 × 10 grid model effective in building understanding for initial decimal ideas? The model created strong mental images that allowed students to build meaning for these numbers, judge the relative sizes of decimals, and understand decimal equivalence. Consider the following examples from student interviews. To see if students did create mental images of the grid, they were asked to only imagine the grids, not actually use them to solve the problems. In the first two examples, notice that students not only pictured the grid but also relied on color to make sense of the task.

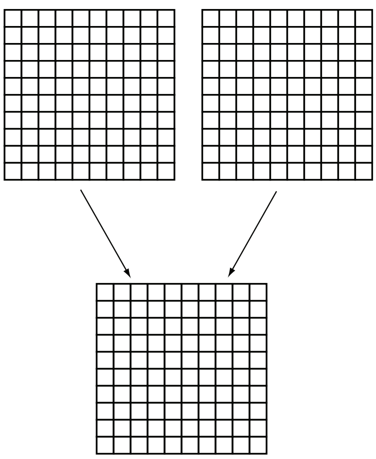
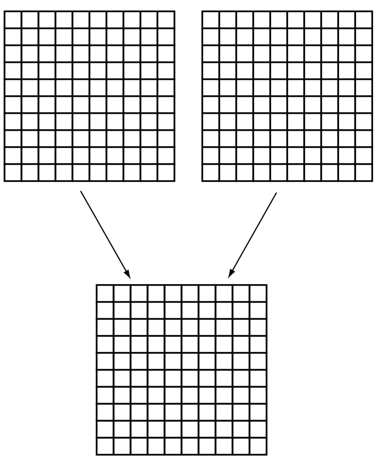
I: Which is bigger, 75 hundredths or nine-tenths? What do you picture when you see the 75 hundredths?
S: I picture seven orange [strips] and five yellow squares.
I: And the tenths?
S: I see nine orange [strips].
I: So then, which is bigger?
S: Nine-tenths.
I: Which is bigger, nine-tenths or 75 hundredths?
S: Nine-tenths would be greater than 75 hundredths. I see nine orange bars and seven orange bars and five yellow squares.
I: Which is bigger, five-tenths or 55 hundredths?
S: I see five-hundredths and half of a square since there are ten thousandths in each square and five is half of it. And I picture one-half of a whole square, the grid. Five-tenths is bigger.

FIGURE 5

Sample problems show the use of the decimal +/– board.

Addition and Subtraction with 10 × 10 Grids

Use a grid to solve each problem. Record the problem with symbols showing your final answer.

Picture	Problem	Estimate	Symbols
	0.75 + 0.02 =	Greater than 1 or less than 1?	
	0.75 – 0.4 =	Greater than $\frac{1}{2}$ or less than $\frac{1}{2}$?	

The 10 × 10 grid has its strengths, but it also has a weakness. Test data with the first group of students showed that the 10 × 10 grid used as a model for addition and subtraction was less effective than the team had hoped in helping students overcome whole-number thinking when adding and subtracting decimals symbolically. For example, when asked to solve 0.37 + 0.4 on a written test, only eleven of twenty-five students answered correctly. Another eleven students resorted to whole-number thinking and answered the problem as 0.41.

When team members considered reasons for this mistake, they first concluded that the use of symbols had been introduced too quickly. Many students needed more time adding and subtracting decimals with a model and needed additional experiences translating their actions with a model to symbols before working solely with symbols. The team made these revisions to the lessons before the next teaching experiment.

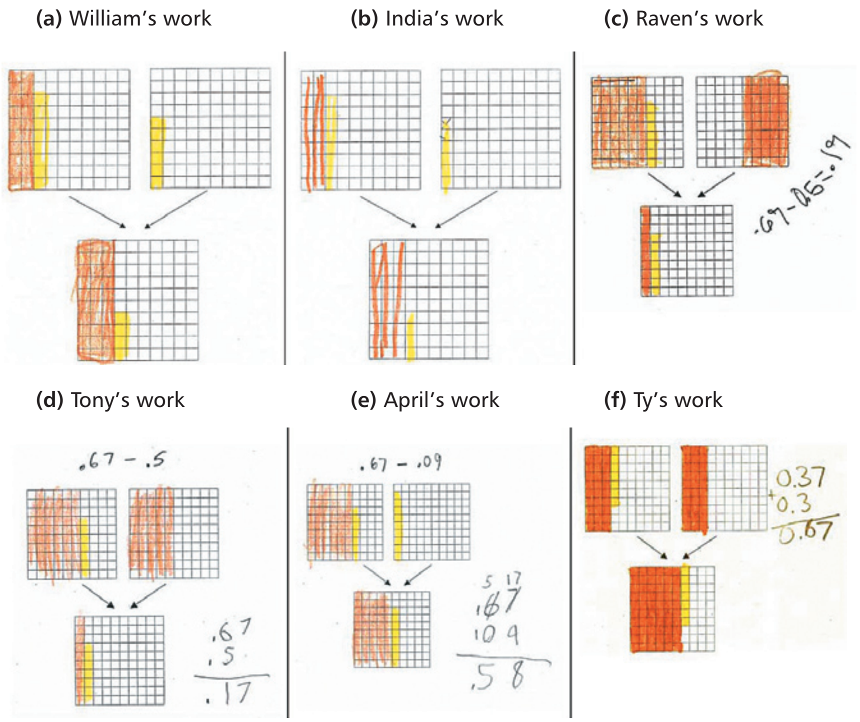
A decimal +/– board

The RNP group considered that the 10 × 10 grid model might need to be adapted to more explicitly show the action of addition and subtraction. Team members developed a new model, a decimal +/– board. This model (see **fig. 5**) enables students to depict each decimal being added or subtracted by using the 10 × 10 grids at the top; they show the final answer on the bottom grid.

Student interviews found that students used this model in a variety of ways. To solve 0.28 +0.06 using the Decimal +/– board, William (all names are pseudonyms), a sixth grader in the second teaching experiment, first showed

FIGURE 6

This student work illustrates how children can act on two numbers either mentally or physically by writing on decimal + /– boards.



0.28 on the left grid as two orange strips and eight yellow squares (see **fig. 6a**). He showed 0.06 as six yellow squares on the right grid. He then said, "I take two of those [yellow squares] to make [a] whole, three-tenths. Then if you minus two from these [six yellow squares], you have four left." William showed the final answer on the bottom grid as three orange strips and four yellow squares. Notice that William mentally operated on the display of the two numbers to determine how to represent the answer on the bottom grid.

In the next example, the student solving the same addition problem physically acted out the addition from the two top grids and then showed the result on the bottom grid (see **fig. 6b**). To solve 0.28 + 0.06 using the board, India first showed 0.28 on the left grid as two orange strips and eight yellow squares. She showed 0.06 as six yellow squares on the right grid. She then crossed off two of the yellow squares from six-hundredths. She shaded three orange strips and four yellow squares on the bottom grid: "This is twenty-eight, so I shade in two orange [strips] and eight yellow [squares]. Then there's only six-hundredths, so shade in six, then take away two. And add them to that. I am going to make tenths so I can make three. And there is still four left over."

Our lessons did not explicitly suggest a way to use the board to model subtraction. The goal of the subtraction lessons was to offer the students the board and see how they would use it to model subtraction.

In the third example (see **fig. 6c**), Raven acted out the subtraction as take-away, marking off the needed number of tenths on the grid on the left side. To solve 0.67 – 0.5 using the board, she first showed 0.67 on the left grid as six orange strips and seven yellow squares. She showed 0.5 as five orange strips on the right grid. She took away five-tenths by crossing out five orange strips on the left grid and then shading the answer on the bottom grid as one orange strip and seven yellow squares. "And I take the five orange [strips] away from this, and I cross them out over here, and then I know when I come down over here [that] I have one-tenth and seven-hundredths."

Tony showed the two amounts on the board but mentally subtracted five-tenths and then showed his final answer (see **fig. 6d**) on the bottom grid: "OK, so you would subtract six bars from five bars, which would give you one left; and there's seven yellows, seven squares, and no squares. So you would just put these squares over there, and there you go."

When asked what his final answer was, Tony stated, "Seventeen-hundredths." Notice that he rewrote the problem vertically to show his work with the grid. He lined up the five-tenths under the six-tenths without having to rewrite 0.5 as 0.50.

To solve 0.67 – 0.09 using the board, April first showed 0.67 on the left grid as six orange strips and seven yellow squares (see **fig. 6e**). She showed 0.09 as nine yellow squares on the right grid: "For this one, you have to subtract seven from nine, which I am going to just use a bar [to do]; and you would add a little circle there and get rid of one of the bars. So, it would be five bars. And then go 1, 2 … 8."

She then shaded five orange strips and eight yellow squares on the bottom grid. "That would be 58 hundredths."

April reflected on the two representations for the numbers involved in the problem. She imagined taking away one-tenth and leaving one yellow square. (She pointed to the grid on the left while explaining this). After working through the problem, she showed that she could also solve the problem by using symbols.

These students used the grids in different ways to solve these problems. The most important part of the grid is that it allows students to clearly represent the two numbers in the problem. Students then acted on the two numbers either mentally or physically by writing on the boards. The bottom grid shows the change after the action. When students used only one 10 × 10 grid to show both decimals, they seemed to lose track of the second number in the problem. Perhaps they were not able to mentally keep track of an image for both numbers when using one grid.

The decimal board seemed to help students translate to symbols accurately while avoiding whole-number mistakes. The RNP group members noticed students reflecting on the numbers represented on the two grids as they wrote the problem using symbols. Students then checked their answer with symbols against the display in the bottom grid.

In another variation, some students used the board to help them set up the problem symbolically; they did not use the bottom grid but solved the problem symbolically after showing the two numbers in the two grids at the top of the board. They reflected on the two numbers on the top part of the board and then correctly recorded the problem using symbols.

The most important part of the grid is that it allows students to clearly represent two numbers.

In Ty's class work (see **fig. 6f**), he did not make the whole-number error of misaligning the numbers despite that fact that the instructors had not introduced the rule to "line up the decimal points." Once the problem was written in symbols, Ty solved the problem symbolically and then recorded his answer using orange and yellow colors on the bottom grid.

The grid helped many students write the problem accurately in a symbolic form that made the written procedure easy to implement. Students then translated the symbolic answer back to the picture.

Implications and reflections

The students' ability to move from one representation of decimals to another aided them in solving addition and subtraction problems. Students were encouraged throughout the lessons to use verbal language that included *tenths*, *hundredths*, and so on. They were also encouraged to use language that stressed the colors that coincided with these values on the decimal +/– boards. More than 90 percent of the students in the second teaching experiment were able to add and subtract correctly using the decimal +/– board on a written test.

Students were also given the opportunity to add without the decimal +/– board, and 87 percent of them were able to correctly add 5.006 + 12.8 without the aid of the board on this same test. This type of problem had not been specifically taught, so it represents the students' ability to use the mathematics they had developed with the different representations and move to purely symbolic reasoning.

In the revised lessons, more time was spent developing students' decimal understanding with the 10 × 10 model and the decimal +/– board after it became clear that additional practice was crucial.

A third model

Students were then challenged to use their understanding of decimals, the 10 × 10 grid, the decimal +/– board model, and their decimal language to represent decimals with a third model, the number line. This model became an additional translation. Students would use the number line to reinforce their work with symbols. Students were asked to translate from the decimal +/– board to the number line and back. They looked at real-world problems and symbolic problems and showed them on the number line. This work with the number line added another dimension to the curriculum. It extended students' thinking by asking them to represent their understanding in a different way.

The RNP group was reminded of the importance of allocating enough time with appropriate models to allow students to develop initial decimal ideas. Sixth graders in this study needed such experiences to build meaning for decimals and operations with decimals. The initial decimal ideas and the procedures for addition and subtraction described in this article are included in *Curriculum Focal Points* (NCTM 2006) goals for grade 4 and grade 5, respectively. Although this article focuses on work with sixth-grade students, using the 10 × 10 grid and the twist on the decimal model can help younger students in grades 4 and 5 meet these decimal goals.

Reflect and discuss: "Models for initial decimal ideas"

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms—and then analyzing and evaluating this information—we identify and explore our own practices and underlying beliefs.

The following questions related to "Models for initial decimal ideas" by Kathleen A. Cramer, Debra S. Monson, Terry Wyberg, Seth Leavitt, and Stephanie B. Whitney are suggested prompts to aid you in reflecting independently on the article, discussing it with your colleagues, and using the authors' ideas to benefit your own classroom practice.

1. You want students to be flexible in how they understand the meaning of decimals. How do your students use color to represent tenths and hundredths on a 10 x 10 grid? Are students able to go back and forth naming, for example, 0.27 as 27 hundredths (all yellow) or two-tenths and seven-hundredths (2 orange strips and 7 yellow squares)?

2. How do your students make use of the decimal +/– board to subtract decimals? Are they concrete, and do they actually write on the board to solve problems, or do they use the board to help them visually keep track of the decimals but actually do the work symbolically?

3. How do your students use the 10 x 10 grid to overcome whole-number thinking when they order decimals?

Tell us how you used "Reflect and discuss" as part of your professional development. Submit letters to *Teaching Children Mathematics* at **tcm@nctm.org**. Include "readers exchange" in the subject line. Find more information at **tcm.msubmit.net**.

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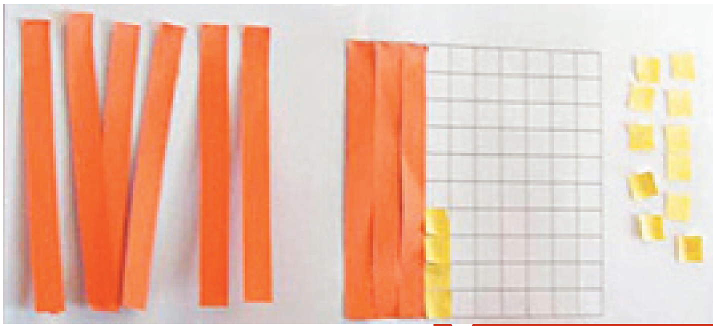

KATHLEEN A. CRAMER

The poster on a white board shows orange strips to represent tenths and yellow squares to represent hundredths.

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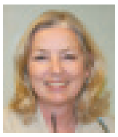
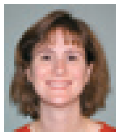
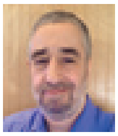
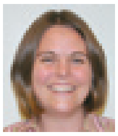
This material is based on work supported by the National Science Foundation under grant number 0628005. The opinions expressed are not necessarily those of the supporting agency, and no endorsement should be inferred.



Kathleen A. Cramer, crame013@umn.edu, is an associate professor at the University of Minnesota and the Project Director for the Rational Number Project (RNP). Her research interests focus on the teaching and learning of fractions, decimals, and proportionality.



Debra S. Monson, stra0042@umn.edu, taught mathematics at Harding High School in St. Paul, Minnesota. She is currently a doctoral student at the University of Minnesota and is interested in mathematics education for preservice elementary teachers. **Terry Wyberg, wyber001@umn.edu**, is a lecturer and coprincipal investigator on the latest RNP grant at the University of Minnesota. His research interests include the teaching and learning of rational number. **Seth Leavitt, leavitt@mpls.k12.mn.us**, is a middle school mathematics teacher in Minneapolis who is interested in helping math teachers secure the best teaching resources available. **Stephanie B. Whitney, whit0818@umn.edu**, taught middle school and high school mathematics in Bloomington. She is currently a doctoral student at the University of Minnesota and is studying ways to help English Language Learners in the mathematics classroom.