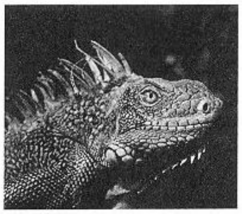

DEPARTMENTS

Technological Tools

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THE DIGITAL STOPWATCH AS A SOURCE OF RANDOM NUMBERS

Ecologists commonly need a way to quickly generate a random digit to make a decision during fieldwork. Random number tables may be brought into the field, a hand-held calculator may be programmed with a simple linear congruential generator, or a die may be tossed. It does not appear to be widely known that many, if not most, ecologists carry an effective random number generator on their wrists: the stopwatch found on most digital watches. The purpose of this article is to demonstrate how digital stopwatches may be used to generate random digits, and to supply some basic guidelines for their use.

A stopwatch may be used to generate random digits as follows: (1) keep the stopwatch running during work; (2) whenever a random digit is needed stop the watch and read the chosen digit (e.g., the digit in the 0.1 second place); (3) restart the watch. An ecologist doing fieldwork may read a stopwatch for random numbers many times in a day. The time between readings will have some mean value, and also an uncertainty dependent on the untidiness of doing things outside. The stopwatch will be an effective random number generator when the uncertainty of the time between readings is large relative to the recurrence interval of the digits chosen for random numbers. The key to efficient use is to choose a digit with a short recurrence interval.

Why does this work? The method is closely analogous to a thought experiment, due to Bradley Efron, that has motivated recent work on the meaning of randomness (Kolata 1986). A dart thrown at a wall painted with 10-foot-wide black and white stripes will land on either black or white predictably (unless one's aim is unusually bad). However, a dart thrown at a wall painted with 1/10-inch-wide stripes will land randomly on black or white. Using a digit with a short recurrence interval is analogous to painting the wall with narrow stripes, and the uncertainty in the time between readings is analogous to the quality of the dart-thrower's aim. Both situations are random for the same reason: there is no practical way to control conditions so that the outcome of any particular instance can be predicted. Fig. 1 illustrates the theory. A probability density function $f(t)$ describes the distribution of times t, between consultations of the stopwatch. The recurrence time r of a particular digit d, is shown by the bracket.

The area of the black stripes is $p(d)$, the probability of occurrence of d. The expression for $p(d)$ can be written:

$$p(d)=\sum_{i=1}^{n}\int_{t(d)_i}^{1+t(d)_i} f(t)dt$$

where t is in the same time units as the chosen digit d (e.g., 0.1 seconds), r is the recurrence interval (e.g., r for 0.1 seconds is 10×0.1 second), $t(d)_i = r(i-1)+d$, and n is chosen such that the integral of $f(t)\,dt$ over the interval $[0, nr]$ is 1.0. The following numerical and practical examples will illustrate some of the principles and pitfalls.

Consider times between consultations distributed as a normal random variable, with mean μ and standard deviation σ. If $\mu = 200$, $n = 40$, and $r = 10$, the values of $p(d)$ for the digits

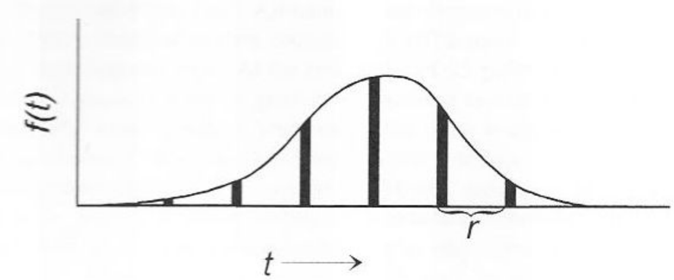

Fig. 1. Probability density function of the distribution of times, t.

Table 1. Values of $p(d)$ for digits 0–9.

Digit	$\sigma = 2$	$\sigma = 4$	$\sigma = 6$	$\sigma = 8$
0	0.19147	0.10795	0.10015	0.10000
1	0.14991	0.10491	0.10009	0.10000
2	0.09205	0.10000	0.10000	0.10000
3	0.04517	0.09502	0.09991	0.10000
4	0.0214	0.09205	0.09985	0.10000
5	0.0214	0.09205	0.09985	0.10000
6	0.04517	0.09502	0.09991	0.10000
7	0.09205	0.10000	0.10000	0.10000
8	0.14991	0.10491	0.10009	0.10000
9	0.19147	0.10795	0.10015	0.10000

0 ... 9 are shown in Table 1 for several different σ.

For $\sigma = 2$ the probabilities are far from the desired 0.1, but the situation improves rapidly as σ approaches the recurrence interval. The normal distribution, being symmetric, is particularly "well behaved" in this situation. Expectations for some extremely skewed distributions were also calculated, and the probabilities for the different digits were not acceptable until σ was a multiple ($\times 4 - \times 15$) of r. Autocorrelation in t is another potential complication, but a stopwatch will still generate acceptable random numbers if σ of the uncorrelated error term is large enough in relation to r. Both of these considerations argue for selecting a digit with the smallest possible time unit, but the accuracy of the stopwatch is also a factor. A stopwatch may spend, for example, slightly less time displaying the 9 than the 6 in the 1/100-second place. Watches should be tested to find the smallest accurate digit; the frequency distribution of digits in a trial run may be used for this.

To test the practical use of the stopwatch technique, the following experiment was carried out. I used the period of time required to walk 40 steps (right foot down) as the interval between stopping the watch. One hundred replicates were taken while walking through a suburban neighborhood over a variety of surfaces from roads to woodland trails. Walking was uninterrupted between stops to record data. The watch was a digital "G-shock" model by Casio, with a stopwatch capable of recording time to a nominal precision of 0.01 second. The distribution of t was approximately normal, with mean = 43.87 seconds, standard deviation = 1.29. There is evidence of weak first-order autocorrelation in t [$\rho(t_i, t_{i+1}) = 0.179$, $P = 0.078$], probably due to the effect of different walking surfaces. Both 0.1 second and 0.01 second digits appear to be effective random number generators in this setting. Neither digit shows first-order (d_i, d_{i+1}) or second-order (d_i, d_{i+2}) autocorrelation, and a χ^2 test shows the overall frequency of digits 0 to 9 is within random expectation.

The stopwatch method does not appear to be widely known among field workers (although at least one Rice University student has re-invented it ad hoc). There are some precautions that need to be kept in mind during use, but these are no more onerous than the precautions accompanying any other method of random number generation. The digital stopwatch is a simple, practical tool to generate random numbers in the field.

Literature cited

Kolata, G. 1986. What does it mean to be random? Science **231**:1068–1070.

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