Number Theory Project
Summer 2016
7th - 9th Grade
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Executive Summary:

Number Theory is an important part of middle level mathematics and sets the stage for high school, college, and even future careers. Students with a solid foundation of how numbers interact and relate to each other are more apt to have greater achievements in mathematics.

The following units include many activities that help students with adding, subtracting, multiplying, and dividing integers. These activities allow students to explore scientific notation to help them have a better understanding of why a number is written the way it is and what it really means and represents. Other activities help students see a relationship between numbers. Students will also find and see the relationship between rational and irrational numbers. The students explore square roots and what it means to be rational or irrational.

The following units and activities are not meant to be taught consecutively; they may work as activities before or after your units they follow in the curriculum you teach from, or they may be used as fun activities at anytime. All of the following lessons and activities have the Minnesota State Standards that they meet at the beginning of the lesson. All activities are meant to be in small groups or partners.
Table of Contents:

Unit 1: Rational and Irrational Numbers p.3
   Day 1: Pre Test and Cryptarithms
   Day 2: Sets / Subsets of Real Numbers
   Day 3: Writing Rational Numbers
   Day 4: Irrational Numbers
   Day 5: Post Test and Cryptarithms

Unit 2: Adding, Subtracting, Multiplying & Dividing Integers p.12
   Day 1: Pre Test / Adding and Subtracting with Hot and Cold cubes
   Day 2: Adding and Subtracting Integers
   Day 3: Creating "Rules" for Adding and Subtracting Integers
   Day 4 & 5: Multiplying and Dividing Integers / Post Test

Unit 3: Scientific Notation p.21
   Day 1: Pre Test / Powers of 10 Video / What is Scientific Notation
   Day 2: Writing in Scientific and Standard Notation
   Day 3: Multiplying and Dividing Scientific Numbers
   Day 4: Multiplying and Dividing Scientific Numbers
   Day 5: Post Test and Sum Game using #'s 1-9

Unit 4: Fractions p.26
   Day 1: Sugar Packet Fun
   Day 2: Fraction Bowling
   Day 3: Tournament of Fractions
   Day 4: Base Ten Block fractions
   Day 5: Chex Mix Math
   Day 6: Street Pavers
   Day 7: How many people can we feed?
   Day 8: Build your own problem
Unit: Real Numbers (5 DAYS)

MN State Standard:
8.1.1.1 Classify real numbers as rational or irrational. Know that when a square root of a positive integer is not an integer, then it is irrational. Know that the sum of a rational number and an irrational number is irrational, and the product of a non-zero rational number and an irrational number is irrational. For example: Classify the following numbers as whole numbers, integers, rational numbers, irrational numbers, recognizing that some numbers belong in more than one category.

8.1.1.2 Compare real numbers; locate real numbers on a number line. Identify the square root of a positive integer as an integer, or if it is not an integer, locate it as a real number between two consecutive positive integers.

MCA III Questions:

The number \( \sqrt{3} \) is located between 2 consecutive integers. Plot the location of the 2 integers. Select the 2 integers you want to choose.

Which expression results in a rational number?

- A. \( 1.5 + \sqrt{1.5} \)
- B. \( 12 - \sqrt{12} \)
- C. \( \frac{3}{4} \cdot \sqrt{\frac{3}{4}} \)
- D. \( 25 \div \sqrt{25} \)
DAY 1 (Pre-Test and Cryptarithms)

LAUNCH: Ask students, “How do teachers know if students are learning or not?” Talk about pre and post tests.

EXPLORE: Describe cryptarithms and put examples on the smart board for students to complete when they finish the pretest. Have students take pretest. (about 20 minutes)

SHARE: Question students about the pre-test and what they think they will be learning. Talk about cryptarithms.

ASSESSMENT: Correct pre-test and reflect from there.
Place a point on the number line given for each of the following irrational numbers.

1. Point A: $\sqrt{2}$
2. Point B: $\sqrt{17}$
3. Point C: $\sqrt{11}$
4. Point D: $\sqrt{8}$
5. Point E: $\sqrt{5}$

A worker in a silver mine descends 57 feet. Use an integer to represent the change in the worker's position.

Which of the following is an irrational number?

○ a) $-236$
○ b) $1.0452...$
○ c) $67$
○ d) $\frac{31}{57}$

Which of the following is a real number?

○ a) 24
○ b) $\frac{21}{65}$
○ c) 3.4
○ d) $\sqrt{625}$
○ e) All of the above.

What are the two whole numbers that are closest to $\sqrt{135}$?

What are the two whole numbers that are closest to $\sqrt{37}$?
How do you describe a rational number?

What does it mean for a decimal to repeat?

What does it mean for a decimal to terminate?

What are decimals called that don't terminate or repeat?

If a rational number has 4 in the denominator, do the decimals repeat, terminate or sometimes repeats/terminates?

If a rational number has a 7 in the denominator, do the decimals repeat, terminate or sometimes repeats/terminates?
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DAY 2  Classifying Real Numbers

LAUNCH: Lay out stacking cups on one desk per group. Have students play with and put the stacking cups together and ask the question: “Can you come up with a real life scenario where the small cup fits into the next, into the next, etc.?”

Example: Little cup: you, Next cup: your family, Next cup: your family fits into your house, Next cup: Your house fits into the town, Next cup: Your town fits into the state.

EXPLORE:

1. ) Give students in their groups about 15 minutes to come up with an example. Pick three groups to share their examples.
2.) Then have students come up with a math example (using three or four cups), this might be difficult. (10 mins)
3.) Write on board: Counting Numbers, Whole Numbers, Integers, Rational Numbers. Ask students to see if they can classify the sets of numbers using the stacking cups. (10-15 minutes).

SHARE:

1.) Have each group present their classification using the stacking cups describing why they classified each cup the way they did. (15-20 mins)

SUMMARIZE:
Label teacher’s set of cups with counting, whole, integer and rational numbers. Question students why the whole numbers don’t “fit” into the natural numbers. Ask students why the “integer” cup does not fit into the whole or natural cup. Continue to question students how the different sets work together or don’t fit together.

ASSESSMENT: Ticket out the door: Write three complete sentences explaining your understanding of natural, whole, integer and rational numbers and how they relate with each other.
DAY 3 (WRITING RATIONAL NUMBERS)

LAUNCH: Have a group describe what the stacking cups represent. Have a student go to board and write out the set of Counting Numbers, Whole Numbers, Integers (may need help), and what the set of Rationals represent.

    Today we are working on Rational numbers.
    1. How can we write them? (Integers) a/b iff a, b elements of integers
    2. Talk about ½, 2/6, 3/6, 4/6, ¾, 6/6.
    3. Can we use decimals? What happens with decimals (terminating (remainder of zero), repeats, sometimes repeats or terminate or infinite)

EXPLORE: Have students in their groups come up with 10 different rational numbers as fractions and their decimal representations and explain why they are rational numbers.

    CHECK FOR CALCULATORS ERRORS, and decimals in either numerator/denominator.

Then ask students if we could figure out a way if we could determine if a decimal repeats, terminates or sometimes repeats & or stops.

EXPLORE: Write #’s 2 - 40 on the board. Have students write T (terminate), R (repeat), S (sometimes). Explain that the numbers represent the denominator.

SUMMARIZE: As students finish, ask them if they see a pattern.

    BASE 10 = 2 * 5
    Terminate: Only 2’s or 5’s or both (nothing else)
    Repeats: no 2’s or 5’s in prime
    Sometimes: has to have at least (1) 2 or 5 and one factor not equal to 2 or 5

Talk about calculator errors. Only using integers in the fractions. How to use the repeating sign. How do we know if they terminate or the calculator rounded the number?

ASSESSMENT: Write on the board: .3333 + .3333 + .3333 = 1. Ask students to come with their reason as to why this is true. (May need a hint to use fractions) ~ Launch into the next day writing fractions from decimals.
DAY 4  (IRRATIONAL NUMBERS)

LAUNCH: Have a few students write their responses to the .333 + .333 + .333 = 1. Then tell the story of building a fence for my cows. The first fence I built was small and area of 100 square feet. Ask the students what each length side of the fence would have to be.

That wasn’t big enough so I needed to build a fence with an area 900 square feet. What would the length of the sides be?

Well the next fence I had to build was for dog and had an area of 137 square feet. What would the sides lengths be?

Talk about irrational numbers. What makes an irrational number? Square roots and estimating the perfect square before and after the irrational number we are working on. Give an example on the board (root 46)

EXPLORE: Put task cards around the room and hand out worksheet to each student. Have groups move around room until they have completed all task cards.

Here is the link to Teachers Pay Teachers where you can buy this activity. 
Square Root Approximations on a Number Line - Task Card Activity (8.NS.2)
**SUMMARIZE:** Ask students which task cards were the most difficult/easy. What is a good way to remember if the real number is a perfect square or irrational?

**ASSESSMENT:** Correct activity worksheet.

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**DAY 5 (Post Test and Cryptarithms)**

**LAUNCH:** Put two problems from each day to have students explain up at the board. Question students about any questions they may have.

**EXPLORE:** Have students take Post Test (the same as the PreTest). Have Cryptarithms on the board for students to complete when they are done with the Post Test

**SUMMARIZE:** Ask students how they thought about the Post Test compared to the Pretest.

**ASSESSMENT:** Correct Post Test and compare to Pre Test and have comparison numbers available for students. REFLECT ON STUDENT LEARNING!
The Chef’s Hot and Cold Cubes
Adding, Subtracting, Multiplying & Dividing Integers
(Interactive Mathematics Program: Shorelineschools.org)

MN State Standards: Numbers and Operations
7.1.2.1 - Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions, and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

Objective: Students will be able to add, subtract, multiply, and divide positive and negative numbers.

MCA Sample Questions Addressed:
Four points are graphed on a line.

-4 -3 -2 -1 0 1 2 3 4

Which point is located at the opposite of \(-2\)?

A. Point J
B. Point K
C. Point L
D. Point M
Day 1: Adding and Subtracting Integers

Materials:
- Red and blue linking cubes
- Printed out cauldrons for each group/student
- Post-it notes
- Hot & Cold cube pre/post “mini opportunity”

Launch: Give students the pre/post “mini opportunity” to see what prior knowledge they are bringing into this unit.

Then head into the story of the famous Chefs.
In a far-off place, there was once a team of amazing chefs who cooked up the most marvelous food ever imagined.

They prepared their meals over a huge cauldron, and their work was delicate and complex. They frequently had to change the temperature of the cauldron to bring out the flavors and cook the food to perfection.
They adjusted the temperature by adding either special hot cubes or special cold cubes to the cauldron or by removing some of the hot or cold cubes that were already in the cauldron.

The cold cubes were similar to ice cubes, except they didn’t melt. The hot cubes were similar to charcoal briquettes, except they didn't lose their heat.

If the number of cold cubes in the cauldron was the same as the number of hot cubes, the temperature of the cauldron was 0 degrees on their temperature scale.

For each hot cube put into the cauldron, the temperature went up 1 degree. For each hot cube removed from the cauldron, the temperature went down 1 degree. Cold cubes worked the opposite way. Each cold cube put in lowered the temperature 1 degree. Each cold cube removed raised it 1 degree.

The chefs used positive and negative numbers to keep track of the temperature changes.

**Explore:**
Have students work in small groups of 3-4 students. Each group should have a set of hot and cold cubes (red and blue linking cubes). One person in the group should be the recorder and write down the arrangements of the cubes (H for hot and C for cold), while the other people in the group will be manipulating the cubes.

Get the students started with the following question:
- The chefs are starting with a cauldron that is at a temperature of 0. How can the chef make the temperature 2 degrees warmer? How many ways different ways can you find?
- Let students see that you could use 2 hot cubes (H H) or you could have many variations of H’s and C’s when they are added in (HC) zero pairs.
- Then have the students see what how many different ways they can have the cauldron be a temperature of -2.
- Continue giving the students different positive and negative numbers to make with their hot and cold cubes until you see that ALL students can make any number in more than one way. Being able to add a (HC) zero pair is important!

**Share:**
After each number is made, have one student from each group go up to the board and show the way they found the number 2, -2, and any other example numbers you gave them. Give them the stipulation that if their example is put up on the board they need to ask their group for help, to come up with a different way. All groups need to be prepared with more than one way to make each number.

Summarize:
Before today, you may have thought that 2 and -2 could only be written one way. You have now learned that numbers can be written multiple ways and still be the same number. On your post-it note, I want you to write your initials on it and show me using H for hot and C for cold two different ways to make the number -5. When you are done, please put your post-it note on the front board, on your way out of class.
Day 2: Adding and Subtracting Integers

Materials:
- Red and blue linking cubes
- Printed out cauldrons for each group/student

Launch:
Yesterday I told you all about some amazing chefs that cooked the most amazing food ever imagined. The only thing is that the temperature has to be JUST RIGHT in the cauldron in order for the food to taste perfect. Today, we are going to help the chefs out by getting the temperature of the cauldron to be just right by adding and taking away Hot and Cold cubes. Let’s try to make wild rice soup together, using the Chef’s famous recipe! The Chef’s put in 8 Hot cubes and then add in 3 Cold cubes. Show this on your cauldron and then have each student or a member from each group show it on the board using H for Hot and C for Cold. Discuss what operation the Chef is performing and how would we write this out as a number sentence?

Explore:
In your groups, figure out what the number sentence would be when you have 3 Cold cubes in your cauldron and you take away 8 Hot cubes. One member from each group will go to write their answer up on the board and then we will have a quick class discussion to make sure all agree and any questions may be answered. If students seem to understand what is going on, release the groups to work on 4 problems together, showing how to solve them using Hot and Cold cubes. If students need more full class discussion time, do the examples all together just as the one before.

A. +4 + -10 = ?
B. +3 - (-2) = ?
C. +4 - (-3) = ?
D. -6 + -4 = ?

Share:
Have each group show their work for each of the 4 problems and discuss how we can find make the same problem using Hot and Cold cubes, in different ways. Ask groups if they can come up with possible “rules” when adding and subtracting positive and negative numbers.

Summarize:
We can now start to correlate that Hot cubes are positive and Cold cubes are negative as we add and subtract our cubes.
Day 3: Creating “Rules” for Adding and Subtracting Integers

Materials:
- Red and blue linking cubes
- Printed out cauldrons for each group/student

Launch:
The last two days we have been adding and subtracting using Hot and Cold cubes as positive and negative numbers. Today, we are going to look for patterns in our number sentences.

Explore:
Have students work in groups to solve the following 4 problems using Hot and Cold cubes if still wanted/needed by students and then discuss what patterns students notice. Repeat with the 4 subtraction problems.

A. 5 + 3 = ?  
B. 5 + (-3) = ?  
C. -5 + (-3) = ?  
D. -5 + (-3) = ?

A. 5 - 3 = ?  
B. 5 - (-3) = ?  
C. -5 - 3 = ?  
D. -5 - (-3) = ?

Share:
Have groups share what patterns they found when adding and then subtracting positive and negative numbers. Try to come up with different “rules” when adding and subtracting. You want the discussion to cover these “rules” and to have full understanding from all students.

(Positive + Positive = ?)  
(Negative + Negative = ?)
(Positive + Negative = ?)  
(Positive - Positive = ?)
(Positive - Negative = ?)  
(Negative - Negative = ?)
(Negative - Positive = ?)

Summarize:
Adding and subtracting positive and negative numbers can be a difficult topic to understand. You can always use red and blue linking cubes to work out the problems.

Day 4 and 5: Multiplying and Dividing Integers

Materials:
- Red and blue linking cubes
- Printed out cauldrons for each group/student
Launch:
The last 3 days you learned how to put smaller amounts of Hot and Cold cubes together to find out what the temperature of the cauldron was going to be. What if the Chefs don’t want to increase or decrease the temperature by such small amounts? Once they get really comfortable in the kitchen, they want to make their recipes for hundreds of people, or they might have a recipe they need to cut down by hundreds. Sometimes they want to raise or lower the temperature by a large amount, but do not want to put the cubes into the cauldron one at a time. So, for large jumps (up or down) in temperature, they would put in or take out a bunch of cubes.

Explore:
Have students work in groups. Students will work through various multiplication and division problems. Start with multiplication on day 4 and go into division on day 5. Multiplying can be considered grouping. Lead through some example problems on the board first, then have the groups solve some together and then put their answers up on the board. Have students still use H for hot and C for cold cubes when they are writing down and explaining their problems.

Share:
Give sample problems to the large group and have them work on the problems with their group. Have each group show their answer up on the board and explain how they got their answer. Make sure students show an answer that has not yet been presented.

A. \(+5 \times +20 = ?\)  
B. \(-3 \times +5 = ?\)  
C. \(-10 \times -5 = ?\)  
D. \(+4 \times -8 = ?\)

A. \(-10 \div -5 = ?\)  
B. \(+4 \div -8 = ?\)  
C. \(+5 \div -2 = ?\)  
D. \(-6 \div +9 = ?\)

Summarize:
On day 5, have students try to come up with “rules” for multiplication and division.
(Positive \(*\) Positive = ?)  (Positive \(/\) Positive = ?)  
(Negative \(*\) Negative = ?)  (Negative \(/\) Negative = ?)  
(Positive \(*\) Negative = ?)  (Positive \(/\) Negative = ?)
Give students the same Hot and Cold cube Pre/Post “mini opportunity” to see how much the students learned and if they need more in depth discussion and exploration.

**Example Word Problems:**
Each of the problems below describes an action by the chefs. Figure out how the temperature would change overall in each of these situations and write an equation to describe the action and the overall result.

a.) Three cold cubes were added and then 5 hot cubes were added.
b.) Five hot cubes were added, and then 4 cold cubes were removed.
c.) Two bunches of 6 cold cubes each were added.
d.) Four bunches of 7 hot cubes each were removed.
e.) Three bunches of 6 cold cubes each were removed.
f.) Five bunches of four cold cubes were removed.
g.) Six bunches of eight hot cubes were added.
h.) Three bunches of five cold cubes were removed.
i.) Nine bunches of two cold cubes were added.
j.) Seven bunches of four hot cubes were removed.
k.) The temperature dropped 18 degrees when nine bunches were added.
l.) The temperature dropped 48 degrees when six bunches were removed.

**Hot and Cold cube Pre/Post “mini opportunity”**

Explain each expression in terms of the “hot and cold cubes” model. Your explanations should describe the action and state how the temperature changes overall.

1. \(-6 + -9\)  
2. \(-7 \, (-10)\)

3. \(+5 * -2\)  
4. \(-4 \, +6\)

5. \(+3 + -7\)  
6. \(-6 * +9\)

7. \(-3 * -4\)  
8. \(+8 \, (-12)\)
9. -12 + +5
Unit: Scientific Notation (5 days of instruction)

MN State Standard:
8.1.1.5 Express approximations of very large and very small numbers using scientific notation; understand how calculators display numbers in scientific notation. Multiply and divide numbers expressed in scientific notation, express the answer in scientific notation, using the correct number of significant digits when physical measurements are involved.

MCA III Question:

Simplify.

- A. $2.5 \times 10^{-2}$
- B. $2.5 \times 10^{-9}$
- C. $2.5 \times 10^{-10}$
- D. $2.5 \times 10^{-11}$
DAY 1: What is Scientific Notation?

Students will complete a 15 question Pretest to measure learning (20 minutes).

LAUNCH:
Writing Numbers in Different Ways:

Present the following information on the SMART board, asking students to say the statements aloud to a partner:
• The population of the world is about 7,117,000,000.
• The distance from Earth to the Sun is about 92,960,000 miles.
• The human body contains approximately 60,000,000,000,000 to 90,000,000,000,000 cells.
• The mass of a particle of dust is 0.000000000753 kg.
• The length of the shortest wavelength of visible light (violet) is 0.0000004 meters.

Discuss the numbers and then assign one or two of them to each pair to rewrite in different ways, such as in words, expanded notation, and powers. Circulate through the room and check for understanding.

Scientific Notation:
Explain to students that there is an easier way to write and say these numbers. Scientific notation is a system developed by scientists and mathematician. (courtesy of http://www.uen.org/mathshorts/downloads/scientific_notation_classroom_activity.pdf)
Then watch Powers of 10 video: Powers of 10 Video

ASSESSMENT: Ticket out the door: Describe two items you thought was interesting from the video that you want to learn more about?
DAY 2 (Writing in Scientific and Standard Notation):
(Launch was from Day 1 video)

LAUNCH: Can 4 groups give an examples from Day 1. Can they write those in scientific notation? Make sure to note that the number in front of the multiplication sign: 1 is less than or equal to x which is less than 10.
   What kind of number has a positive exponent
   What kind of number then has a negative exponent?

EXPLORE: Have students complete the Scientific Notation Task Cards #1-20 converting from Standard and Scientific Notation in their desk groups (making sure not to leave your group behind).

Scientific Notation Operations - 48 Task Cards

SUMMARIZE: Ask students what problems they thought were difficult. Ask students what is tricky about writing and converting numbers from standard notation and scientific notation. How can we remember the rules?

ASSESSMENT: Grade activity cards to make sure students have understanding.
**DAY 3 & 4 (Multiplying/Dividing Scientific Numbers)**

**LAUNCH:** Start with story about money: There are approximately $4 \times 10^{9}$ people in the world. If each person made $3.6 \times 10^{4}$ dollars in a year, how much money was made worldwide?

Go over Multiplying Scientific Notation rules (adding exponents).

Then go over Dividing Rules (subtracting exponents).

**EXPLORE:** Have students in their groups complete the same Scientific Notation activity but cards #21-48.

**SUMMARIZE:** Question students on what problem was hard/easy/tricky. How can we prevent mistakes? Did you have to convert any numbers into correct scientific notation form?

**ASSESSMENT:** Correct the task card activity to make sure students added/subtracted exponents correctly and converted in Scientific Notation.
DAY 5 (Post Test and Sum using #1-9)

LAUNCH: Put 2 problems from each of the previous days on the board and have students explain the process of finding the answer.

Then explain the rules for the “SUM” game adding #‘s 1-9 using each number only once. Students will work on this as they finish the post test.

```
X X X
+ X X X
---------
X X X
```

EXPLORE: Have students complete the Post Test.

SUMMARIZE: Ask students what they thought about the post test compared to the pretest. Ask students to come up to the board to explain their sum answers.

ASSESSMENT: Reflect on pre-post test scores. Make sure to give scores to students
Sugar Packet Fun

7.2.2.3 Use knowledge of proportions to assess the reasonableness of solutions.
7.2.4.2 Solve equations resulting from proportional relationships in various contexts.

Materials
- variety of sports drinks and sodas
- print out of 20 oz soda and sugar packet labels (act 2)

Launch: Show the first video (act 1) which will engage the students into the first conversation. Which should be centered around what is happening in the video and also write down an estimate of how many sugar packets they think are in one 20 oz bottle of soda.

Explore: SW brainstorm what information they will need (tw provide nutrition information about both the 20 oz soda and sugar packets act 2) SW work through setting up proportions and solve for the correct number of packets.

Share: We would share both the amount that each group got along with how they set up the proportion. Show the final video act 3. We will also discuss how the different estimations compared. TW then bring out other sports drinks and sodas to see how they compare. Can even expand to estimating and trying to find what drink or food would have 50 sugar packets in it.

Summarize: TW talk about how the proportions were used and that this process can be used to in other real world activity. Could also lead into a talk about healthy beverage choices.
Fraction bowling

7.1.2.5 Use proportional reasoning to solve problems involving ratios in various context.
7.1.2.1 Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

Materials
- blank dice or set of bowling pins and ball (can get plastic ones for cheap at target)
- fraction bowling worksheet

Launch: TW hand out fraction bowling game boards and explain the instructions.
Students in groups of 2 will take turns rolling two dice then they will find the sum of the two dice (each 6 sided dice with the 6th side blank) and that is how many pins they knocked down. If they roll a black side that is zero. If you have a set of bowling pins you can have the kids take turns and write down the number of pins they knocked down after 2 rolls.

Explore: Students will play the game in paris keeping track of their scores. When they record their score each pair will work together to translate the fraction to a decimal and then to a percent. They can also expand to rolling one dice and then adding the two fractions together.

Share: Once the students have all played through 6 frames apiece have the students in small groups talk about any patterns they can see between the fractions, decimals, and percents. Then the students will share out their findings to the class. If students did not bring up simplification and finding equivalent fractions.

Summarize: TW summarize the different ways that students came to their answers and make sure that the students are understanding the vocab of equivalent fractions and simplification. Also if not brought up during conversation talk about how you can go back and forth between fractions, decimals, and percents.
Fraction Bowling

Fraction ________
Decimal ________
Percent ________%

Fraction ________
Decimal ________
Percent ________%

Fraction ________
Decimal ________
Percent ________%

Fraction ________
Decimal ________
Percent ________%

Fraction ________
Decimal ________
Percent ________%
Tournament of Fractions

7.1.1.4 Compare positive and negative rational numbers expressed in various forms using the symbols $<, >, =, \leq, \geq$.

7.1.1.5 Recognize and generate equivalent representations of positive and negative rational numbers, including equivalent fractions.

Materials
- Fraction flash cards for war

Launch: Randomly assign denominators to each of the students (would say that you should pick numbers between 10 and 40). After the kids each have their denominators they will pick a numerator.
To add to this lesson you could have your students play war using fraction cards to work on simplification and finding equivalent fractions.

Explore: Students will start by comparing their fraction with one other student in the class, who ever has a larger fraction will move forward. Those two students will then add their fractions and the greater of the two on the next bracket. The class will then have the start of a tournament bracket. The students with the larger fractions of the first round will then compare their fraction with the next student in the bracket. As a student progresses in the bracket every student that has a fraction less than theirs now becomes part of their team until their is one greatest fraction.

Share: Students will explain why they think that the winner won and what makes a fraction bigger and how to make the largest fraction they can with any given denominator.

Summarize: TW go over the vocab that was being used (denominator, numerator, equivalent fractions). Also talk through the different ways of finding common denominators that were used.
Base Ten Block Fractions

7.1.1.2 Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.

7.1.2.1 Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

7.1.2.5 Use proportional reasoning to solve problems involving ratios in various contexts.

Materials:
- base ten block sheets
- colored pencils

Launch: Hand out base ten paper and have the kids color in any number of cubes both in the hundreds (flat) and the tens (longs). Ask them about how this could relate to the fraction bowling that they did before.

Explore: In small groups students will talk about how the base ten blocks compare to the fraction bowling activity. What fraction is shown? Can this fraction be reduced or is it in its simplest form?

Q: Jon, Jack, and Jill are all sharing a package of markers. If Jack has $\frac{1}{4}$ of the markers, Jon has $\frac{1}{5}$ of the markers and Jill has the rest what fraction of the markers does Jill have?

SW color in the base ten block pages to complete this 2 step problem.

Expand- Q: If there are 20 markers in the package how many markers will each of the students be have?

Share: SW share their answers with the class and explain how they got to their answer. If possible have students explain it both with the base ten blocks and how they would represent that in a number sentence.

Summarize: TW talk about the steps needed to compete this problem, which has both add and subtraction of fractions. Talk about the vocab of common denominator and have a class definition of that along with equivalent fractions.
Chex Mix Math

7.4.3.2 Calculate probability as a fraction of sample space or as a fraction of area. Express probabilities as percents, decimals and fractions.
7.4.3.3 Use proportional reasoning to draw conclusions about and predict relative frequencies of outcomes based on probabilities.

Materials
- Snack size bags of chex mix for each group or one large bag and just give each group a scoop.
- Chex Mix worksheet

Launch: Hand out chex mix worksheet. Have the students first estimate how many chex mix they have before they start counting them. Then explain the first step of the worksheet # 1-3.

Explore: SW work in groups of 2 counting out their different numbers of each piece of chex mix then turn those into their simplest form. Next the students in pairs will work through the addition and subtraction #4 of the different pieces of chex mix, then tally up their pieces of chex mix. Finally the pairs will group up with another group of 2 to compare their answers 3 of the comparisons are on the worksheet but students can also compare the subtraction results.

Share: Student groups will then share with the class their findings to see if the same trends are throughout all of the bags of chex mix.

Summarize: TW review student’s responses and have a discussion about how the different processes that worked and the different ways students completed the problem. Which part of the chex mix is your favorite part? Who have the most and least common favorite pieces?
1. Estimate: How many Chex Mix do you have? ____________________________

2. Count your Chex Mix and write down the total. ____________________________

3. Fractions: Using your total, write a fraction for each of your Chex Mix shapes.

4. Addition

   _______  +  _______  =  _______

   _______  +  _______  =  _______

   _______  +  _______  =  _______
5. Tally each Chex Mix shape.
6. Compare your Chex Mix with a friend.

Who had the most **pretzels** (squares and circles)? Write down how many each of you had. Circle the number that is greater.

_____________________ had ____________.  ____________________ had ____________.

Who had the most **chex cereal squares**?

_____________________ had ____________.  ____________________ had ____________.

Who had the most **bread sticks / chips**?

_____________________ had ____________.  ____________________ had ____________.
**Street Pavers**

7.1.1.2 Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.

7.1.2.1 Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

7.1.2.5 Use proportional reasoning to solve problems involving ratios in various contexts.

**Materials**
- blocks or legos (enough so each group of students can have 12-20 of two different colors)

**Launch:** Hand out blocks that connect or legos that are all the same size can use different colors one for the paver and one for the road. Introduce the first problem: We have 10 miles of road and our paver can pave only 2 miles per day. How many days will it take to pave our 10 miles of road.

**Explore:** SW look for ways to model this problem with the blocks/legos. After sharing students process of finding the number of days with the manipulatives then TW give problem #2: The construction company now purchased a paver the will pave 3 miles per day and the road it 9 miles long. How many days will it take to pave that new road with our new paver.

Then moving into the next problems sharing between each problem
#3 The road it 5 miles and the paver can pave ½ miles per day.
#4 The road is 6 miles long and the paver can pave ¼ a mile per day.
#5 The road is ½ of a mile and the paver can pave ⅓ of a mile per day.

**Share:** SW share after each problem how they got to their answer. As a teacher if students start wanting to just draw pictures or write it out abstractly then make sure that they can still relate that to the concrete application of the problem.

**Summarize:** Can we as a class come up with a processes that will work everytime? Is there more than one way to complete these problems? Bring vocab into the convocation of common divisors, can relate this to dividing fractions using common divisors.
How Many People Can We Feed?

7.1.1.2 Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.

7.1.2.1 Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.

7.1.2.5 Use proportional reasoning to solve problems involving ratios in various contexts.

Materials:
- individual white boards or white board space for each group of 4 students

Launch: TW hand out white boards and introduce the first question.
Q: Joel has five pieces of wood to construct birdhouses. If each birdhouse takes \( \frac{2}{3} \) of a piece of wood, how many bird houses can Joel make?

Explore: SW work through the first question in their groups. How can you show this problem through a picture? Can this problem be done with just a number sentence? What is the number sentence and can you show where their numbers came from in the word problem?
Q2: Mrs. Smith has 6 cups of dog food left in the dog food bag. If every day Mrs. Smith feeds her dog \( \frac{3}{4} \) cup of dog food, how many day of food does she have left?
Q3: Mrs. Woods is having a party and she has 3lbs of fish. If she wants to feed each person \( \frac{2}{3} \) of a lb of fish, how many people can she feed? (What about the extra half of a serving?)

Share: After there has been work time for question 1 then SW share out their answers and the process they used to get there. Then as a class we will move onto the next question take group work time and then share out about those problems.

Summarize: TW go over the different processes that the students have been using and that we have been drawing pictures to show multiplication of fractions.
Build Your Own Problem

7.1.1.2 Understand that division of two integers will always result in a rational number. Use this information to interpret the decimal result of a division problem when using a calculator.
7.1.2.1 Add, subtract, multiply and divide positive and negative rational numbers that are integers, fractions and terminating decimals; use efficient and generalizable procedures, including standard algorithms; raise positive rational numbers to whole-number exponents.
7.1.2.5 Use proportional reasoning to solve problems involving ratios in various contexts.

Launch: TW give students a number sentence: $4 \frac{2}{3} = 6$. TW ask the students to then come up with a story problem and picture to represent the number sentence.

Explore: SW work in groups of 2-4 to create a story that will need to use the number sentence given to complete the problem.
Other number sentences for kids to make into a story problem.

$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$
$3 \frac{3}{4} = 4$

Share: SW share their stories that they have come up with and show how they would solve those questions. What is similar about these stories? What is different between the stories? How can you show that you are using the correct number sentence and can you translate that back to being able to write a number sentence from any word problem that are given?

Summarize: TW walk through the process that students are using to come up with a story for the given number sentence and about how you can pull a number sentence out of a any given word problem. Go over key words to look for, questions, and locating the numbers, which numbers are needed and which are given but not used.
Pre Test Fractions

Name ________________________________

1

a. Shade the portion of the area of the rectangle that represents 0.725.

b. What fractional part of the area did you shade?

c. What percentage of the area did you shade?

2

a. Shade 3/8 of the area of the rectangle.

b. What percentage of the area did you shade?

c. What decimal number does the shaded portion of the rectangle represent?

For each of the following scenarios, sketch a diagram that shows the mathematical relationships in the situation. List several mathematical questions about the situation that you think you can answer by reasoning from your diagram. Then answer your questions and show or explain your reasoning. Use the back of this sheet if necessary.

3

Jamaal's Snowstorm

After a heavy January snowstorm, the snow in Jamaal's front yard was 42 inches deep, which is \(3\frac{1}{2}\) times as deep as it was before the storm. The amount of new snow that fell during the storm is \(\frac{5}{6}\) of the all-time record for a snowstorm in Jamaal's state.
4 Bill’s Trail Mix factory gives customers the following information. Use the pattern in the table to answer the questions.

<table>
<thead>
<tr>
<th>Grams of Trail Mix</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>150</td>
</tr>
<tr>
<td>150</td>
<td>450</td>
</tr>
<tr>
<td>300</td>
<td>900</td>
</tr>
<tr>
<td>500</td>
<td>1500</td>
</tr>
</tbody>
</table>

a. Fiona eats 75 grams of trail mix. How many Calories does she eat?
b. Rico eats trail mix containing 1000 calories. How many grams of trail mix does he eat?

5 The construction company now purchased a paver the will pave 3 miles per day and the road it 15 miles long. How many days will it take to pave that new road with our new paver?

6. Jon, Jack, and Jill are all sharing a package of markers. If Jack has 1/4 of the markers, Jon has 2/5 of the markers and Jill has the rest. What fraction of the markers does Jill have? Rank who has the greatest number to fewest number of markers.

Post Test Fractions

1

a. Shade 87.5% of the area of the rectangle.
b. What fractional part of the area did you shade?

c. What decimal number does the shaded portion of the rectangle represent?
Bill’s Trail Mix factory gives customers the following information. Use the pattern in the table to answer the questions.

<table>
<thead>
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<tr>
<td>100</td>
<td>900</td>
</tr>
<tr>
<td>300</td>
<td>1500</td>
</tr>
</tbody>
</table>

a. Fiona eats 75 grams of trail mix. How many Calories does she eat?

b. Rico eats trail mix containing 1000 calories. How many grams of trail mix does he eat?
5 The construction company now purchased a paver that will pave 4 miles per day and the road it 24 miles long. How many days will it take to pave that new road with our new paver?

6. Jon, Jack, and Jill are all sharing a package of markers. If Jack has 1/3 of the markers, Jon has 2/5 of the markers and Jill has the rest what fraction of the markers does Jill have? Rank who has the greatest number to fewest number of markers.