4-4  a) \( P(1 < X) = \int_{4}^{\infty} e^{-x/4} \, dx = -e^{-(x/4)} \bigg|_{1}^{\infty} = 1 \), because \( f_X(x) = 0 \) for \( x < 4 \). This can also be obtained from the fact that \( f_X(x) \) is a probability density function for \( 4 < x \).

b) \( P(2 \leq X \leq 3) = \int_{4}^{5} e^{-x/4} \, dx = -e^{-(x/4)} \bigg|_{2}^{5} = 1 - e^{-1} = 0.6321 \)

c) \( P(5 < X) = 1 - P(X \leq 5) \). From part b, \( P(X \leq 5) = 0.6321 \). Therefore, \( P(5 < X) = 0.3679 \).

d) \( P(8 < X < 12) = \int_{9}^{12} e^{-(x-4)} \, dx = -e^{-(x-4)} \bigg|_{8}^{12} = e^{-4} - e^{-8} = 0.0180 \)

e) \( P(X < x) = \int_{4}^{x} e^{-(x-4)} \, dx = -e^{-(x-4)} \bigg|_{4}^{x} = 1 - e^{-(x-4)} = 0.90 \).

Then, \( x = 4 - \ln(0.10) = 6.303 \)

4-6  a) \( P(X > 3000) = \int_{3000}^{\infty} \frac{e^{-x/1000}}{1000} \, dx = -e^{-x/1000} \bigg|_{3000}^{\infty} = e^{-3} = 0.05 \)

b) \( P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-x/1000}}{1000} \, dx = -e^{-x/1000} \bigg|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233 \)

c) \( P(X < 1000) = \int_{0}^{1000} \frac{e^{-x/1000}}{1000} \, dx = -e^{-x/1000} \bigg|_{0}^{1000} = 1 - e^{-1} = 0.6321 \)

d) \( P(X < x) = \int_{0}^{x} \frac{e^{-x/1000}}{1000} \, dx = -e^{-x/1000} \bigg|_{0}^{x} = 1 - e^{-x/1000} = 0.10 \).

Then, \( e^{-x/1000} = 0.9 \), and \( x = 1000 \ln 0.9 = 105.36 \).
4-9 a) \( P(X < 2.25 \text{ or } X > 2.75) = P(X < 2.25) + P(X > 2.75) \) because the two events are mutually exclusive. Then, \( P(X < 2.25) = 0 \) and
\[
P(X > 2.75) = \int_{2.75}^{2.8} 2 \, dx = 2(0.05) = 0.10.
\]
b) If the probability density function is centered at 2.55 meters, then \( f_x(x) = 2 \) for
\( 2.3 < x < 2.8 \) and all rods will meet specifications.

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4-12. a) \( P(X < 1.8) = P(X \leq 1.8) = F_X(1.8) \) because \( X \) is a continuous random variable. Then,
\[
F_X(1.8) = 0.25(1.8) + 0.5 = 0.95
\]
b) \( P(X > -1.5) = 1 - P(X \leq -1.5) = 1 - 0.125 = 0.875
\]
c) \( P(X < -2) = 0
\]
d) \( P(-1 < X < 1) = P(-1 < X \leq 1) = F_X(1) - F_X(-1) = .75 - .25 = 0.50
\]

4-16. Now, \( f(x) = x/8 \) for \( 3 < x < 5 \) and \( F_X(x) = \int_3^x \frac{dx}{8} = \frac{x^2}{16} \bigg|_3^x = \frac{x^2 - 9}{16}
\]
for \( 0 < x \). Then, \( F_X(x) = \begin{cases} 0, & x < 3 \\ \frac{x^2 - 9}{16}, & 3 \leq x < 5 \\ 1, & x \geq 5 \end{cases} \)

4-20. \( f(x) = 2e^{-2x}, \ x > 0 \)

4-26. \( E(X) = \int_3^5 \frac{x}{8} \, dx = \frac{x^2}{16} \bigg|_3^5 = \frac{5^2 - 3^2}{24} = 4.083 \)
\[
V(X) = \int_3^5 (x - 4.083)^2 \frac{x}{8} \, dx = \int_3^5 \left( \frac{x^3}{8} - \frac{8.166x^2}{8} + \frac{16.6709x}{8} \right) dx
\]
\[
= \frac{1}{8} \left( \frac{x^4}{4} - \frac{8.166x^3}{3} + \frac{16.6709x^2}{2} \right) \bigg|_3^5 = 0.3264
\]
4-28. a) 

\[ E(X) = \int_{1200}^{1210} x^2 \, 0.1 \, dx = 0.05 \int_{1200}^{1210} x^2 \, dx = 1205 \]

\[ V(X) = \int_{1200}^{1210} (x - 1205)^2 \, 0.1 \, dx = 0.1 \int_{1200}^{1210} \frac{(x - 1205)^3}{3} \, dx = 8.333 \]

Therefore, \( \sigma_x = \sqrt{V(X)} = 2.887 \)

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

\[ P(1195 < X < 1205) = P(1200 < X < 1205) = \int_{1200}^{1205} 0.1 \, dx = 0.1 \cdot \left[ x \right]_{1200}^{1205} = 0.5 \]