STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Assignment 6
Chapter 3: 53, 59, 61, 65, 66, 67, 79, 81, 85, 86, 89.

3-53. $E(X) = (3+1)/2 = 2$, $V(X) = [(3-1+1)^2 -1]/12 = 0.667$

3-59. The range of Y is 0, 5, 10, ..., 45, $E(X) = (0+9)/2 = 4.5$

$$\begin{aligned} E(Y) &= 0(1/10)+5(1/10)+...+45(1/10) \\ &= 5[0(0.1) +1(0.1)+ ... +9(0.1)] \\ &= 5E(X) \\ &= 5(4.5) \\ &= 22.5 \end{aligned}$$

$V(X) = 8.25$, $V(Y) = 5^2(8.25) = 206.25$, $\sigma_Y = 14.36$

3-61. A binomial distribution is based on independent trials with two outcomes and a constant probability of success on each trial.

a) reasonable
b) independence assumption not reasonable
c) The probability that the second component fails depends on the failure time of the first component. The binomial distribution is not reasonable.
d) not independent trials with constant probability
e) probability of a correct answer not constant.
f) reasonable
g) probability of finding a defect not constant.
h) if the fills are independent with a constant probability of an underfill, then the binomial distribution for the number packages underfilled is reasonable.
i) because of the bursts, each trial (that consists of sending a bit) is not independent
j) not independent trials with constant probability

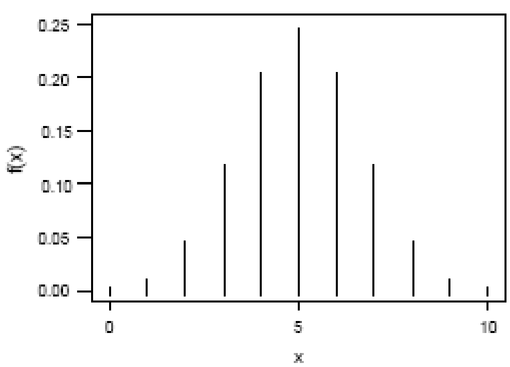
3-65. a) $P(X=5)=\binom{10}{5}0.01^5(0.99)^5 = 2.40\times10^{-8}$

$$b) P(X\le 2)=\binom{10}{0}0.01^0(0.99)^{10}+\binom{10}{1}0.01^1(0.99)^9+\binom{10}{2}0.01^2(0.99)^8$$
$$=0.9999$$

$$c) P(X\ge 9)=\binom{10}{9}0.01^9(0.99)^1+\binom{10}{10}0.01^{10}(0.99)^0 = 9.91\times10^{-18}$$

$$d) P(3\le X<5)=\binom{10}{3}0.01^3(0.99)^7+\binom{10}{4}0.01^4(0.99)^6 = 1.138\times10^{-4}$$

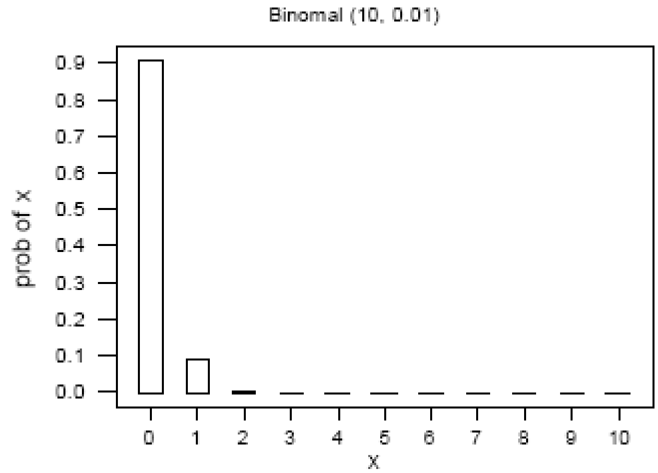
3-66.



a) $P(X=5)=0.9999$, X= 5 is most likely, also $E(X)=np=10(0.5)=5$

b) Values X=0 and X=10 are the least likely, the extreme values

3-67.



P(X = 0) = 0.904, P(X = 1) = 0.091, P(X = 2) = 0.004, P(X = 3) = 0. P(X = 4) = 0 and so forth.
Distribution is skewed with $E(X) = np = 10(0.01) = 0.1$

a) The most-likely value of X is 0.

b) The least-likely value of X is 10.

3-79. Let X denote the passengers with tickets that do not show up for the flight. Then, X is binomial with n = 125 and p = 0.1.

$$a) P(X \geq 5) = 1 - P(X \leq 4)$$

$$= 1 - \left[\binom{125}{0} 0.1^0 (0.9)^{125} + \binom{125}{1} 0.1^1 (0.9)^{124} + \binom{125}{2} 0.1^2 (0.9)^{123} + \binom{125}{3} 0.1^3 (0.9)^{122} + \binom{125}{4} 0.1^4 (0.9)^{121} \right]$$

$$= 0.9961$$

$$b) P(X > 5) = 1 - P(X \leq 5) = 0.9886$$

3-81. a) $P(X = 1) = (1 - 0.5)^0 0.5 = 0.5$

b) $P(X = 4) = (1 - 0.5)^3 0.5 = 0.5^4 = 0.0625$

c) $P(X = 8) = (1 - 0.5)^7 0.5 = 0.5^8 = 0.0039$

d) $P(X \leq 2) = P(X = 1) + P(X = 2) = (1 - 0.5)^0 0.5 + (1 - 0.5)^1 0.5$

$= 0.5 + 0.5^2 = 0.75$

e.) $P(X > 2) = 1 - P(X \leq 2) = 1 - 0.75 = 0.25$

3-85. Let X denote the number of trials to obtain the first successful alignment. Then X is a geometric random variable with p = 0.8

a) $P(X=4)=(1-0.8)^3 0.8=0.2^3 0.8=0.0064$

b) $P(X\le 4)=P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$$=(1-0.8)^0 0.8+(1-0.8)^1 0.8+(1-0.8)^2 0.8+(1-0.8)^3 0.8$$

$$=0.8+0.2(0.8)+0.2^2(0.8)+0.2^3 0.8=0.9984$$

c) $P(X\ge 4)=1-P(X\le 3)=1-[P(X=1)+P(X=2)+P(X=3)]$

$$=1-[(1-0.8)^0 0.8+(1-0.8)^1 0.8+(1-0.8)^2 0.8]$$

$$=1-[0.8+0.2(0.8)+0.2^2(0.8)]=1-0.992=0.008$$

3-86. Let X denote the number of people who carry the gene. Then X is a negative binomial random variable with r=2 and p = 0.1

a) $P(X\ge 4)=1-P(X<4)=1-[P(X=2)+P(X=3)]$

$$=1-\left[\binom{1}{1}(1-0.1)^0 0.1^2+\binom{2}{1}(1-0.1)^1 0.1^2\right]=1-(0.01+0.018)=0.972$$

b) $E(X)=r/p=2/0.1=20$

3-89. p=0.13

(a) $P(X=1) = (1-0.13)^{1-1}*0.13=0.13.$

(b) $P(X=3)=(1-0.13)^{3-1}*0.13 =0.098$

(c) $\mu=E(X)= 1/p=7.69\approx 8$