STAT 3631/5631 Homework

Applied Statistics and Probability for Engineers
Montgomery and Runger

Assignment 10
Chapter 4: 70, 73, 78, 81, 88, 95, 98, 104.

4-70. Let X denote the number of error accounts.
X ~ BIN(362000, 0.001)
(a)E(X) = 362
Stdev(X) =19.0168

(b) $Z = \frac{X - 362000\times 0.001}{\sqrt{362(1-0.001)}} = \frac{X-362}{19.0168}$ is approximately N(0,1).

$$P(X<350) = P(X \le 349+0.5) = \Phi(\frac{349.5-362}{19.0168}) = \Phi(-0.6573) = 0.2555$$

(c) $\Phi^{-1}(0.95)\times 19.0168 + 362 - 0.5 = 392.7799$

(d)

$$P(X>400) = 1-P(X\le 400) = 1-P(X\le 400+0.5) = 1-\Phi(\frac{400.5-362}{19.0168}) = 1-\Phi(2.0245) = 0.0215$$

Then the probability is $0.0215^2 = 4.6225\times 10^{-4}$.

4-73. Let X denote the number of particles in 10 $cm^2$ of dust. Then, X is a Poisson random variable with $\lambda = 10(1000) = 10{,}000$. Also, E(X) = λ = $10^4$ and V(X) = λ = $10^4$

$$P(X > 10{,}000) \cong P(Z > \tfrac{10{,}000.5 - 10{,}000}{\sqrt{10{,}000}}) = P(Z > 0.005) = 0.498$$

4-78. (a) $P(X<5) = 0.3935$

(b) $P(X<15 \mid X>10) = \frac{P(X<15, \quad X>10)}{P(X>10)} = \frac{P(X<15)-P(X<10)}{1-P(X<10)} = \frac{0.1447}{0.3679} = 0.3933$

(c) They are the same.

4-81. Let X denote the time until the first call. Then, X is exponential and $\lambda = \frac{1}{E(X)} = \frac{1}{15}$ calls/minute.

a) $P(X>30) = \int_{30}^{\infty} \frac{1}{15} e^{-\frac{x}{15}} dx = -e^{-\frac{x}{15}}\Big|_{30}^{\infty} = e^{-2} = 0.1353$

b) The probability of at least one call in a 10-minute interval equals one minus the probability of zero calls in a 10-minute interval and that is P(X > 10).

$$P(X>10) = -e^{-\frac{x}{15}}\Big|_{10}^{\infty} = e^{-2/3} = 0.5134.$$

Therefore, the answer is 1- 0.5134 = 0.4866. Alternatively, the requested probability is equal to P(X < 10) = 0.4866.

c) $P(5 < X < 10) = -e^{-\frac{x}{15}}\Big|_{5}^{10} = e^{-1/3} - e^{-2/3} = 0.2031$

d) P(X < x) = 0.90 and $P(X < x) = -e^{-\frac{t}{15}}\Big|_{0}^{x} = 1 - e^{-x/15} = 0.90$. Therefore, x = 34.54 minutes.

4-88. Let X denote the distance between major cracks. Then, X is an exponential random variable with $\lambda = 1/E(X) = 0.2$ cracks/mile.

a) $P(X > 10) = \int_{10}^{\infty} 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_{10}^{\infty} = e^{-2} = 0.1353$

b) Let Y denote the number of cracks in 10 miles of highway. Because the distance between cracks is exponential, Y is a Poisson random variable with $\lambda = 10(0.2) = 2$ cracks per 10 miles.

$$P(Y = 2) = \frac{e^{-2}2^2}{2!} = 0.2707$$

c) $\sigma_X = 1/\lambda = 5$ miles.

d) $P(12 < X < 15) = \int_{12}^{15} 0.2e^{-0.2x}dx = -e^{-0.2x}\Big|_{12}^{15} = e^{-2.4} - e^{-3} = 0.0409$

e) $P(X > 5) = -e^{-0.2x}\Big|_{5}^{\infty} = e^{-1} = 0.3679$. By independence of the intervals in a Poisson process, the answer is $0.3679^2 = 0.1353$. Alternatively, the answer is $P(X > 10) = e^{-2} = 0.1353$. The probability does depend on whether or not the lengths of highway are consecutive.

f) By the memoryless property, this answer is P(X > 10) = 0.1353 from part e).

4-95. a) $\Gamma(6) = 5! = 120$

b) $\Gamma(\frac{5}{2}) = \frac{3}{2}\Gamma(\frac{3}{2}) = \frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{3}{4}\pi^{1/2} = 1.32934$

c) $\Gamma(\frac{9}{2}) = \frac{7}{2}\Gamma(\frac{7}{2}) = \frac{7}{2}\frac{5}{2}\frac{3}{2}\frac{1}{2}\Gamma(\frac{1}{2}) = \frac{105}{16}\pi^{1/2} = 11.6317$

4-98. Let X denote the pounds of material to obtain 15 particles. Then, X has an Erlang distribution with r = 15 and $\lambda = 0.01$.

a) $E(X) = \frac{r}{\lambda} = \frac{15}{0.01} = 1500$ pounds.

b) $V(X) = \frac{15}{0.01^2} = 150{,}000$ and $\sigma_X = \sqrt{150{,}000} = 387.3$ pounds.

4-104. $\Gamma(r) = \int_0^{\infty} x^{r-1}e^{-x}dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$$\Gamma(r) = -x^{r-1}e^{-x}\Big|_0^{\infty} + (r-1)\int_0^{\infty} x^{r-2}e^{-x}dx = (r-1)\Gamma(r-1).$$