4-70. Let $X$ denote the number of error accounts. 
$X \sim \text{BIN}(362000, 0.001)$
(a) $E(X) = 362$
$\text{Stdev}(X) = 19.0168$

(b) $Z = \frac{X - 362000 \times 0.001}{\sqrt{362(1 - 0.001)}} = \frac{X - 362}{19.0168}$ is approximately $N(0, 1)$.

$P(X < 350) = P(X \leq 349 + 0.5) = \Phi\left(\frac{349.5 - 362}{19.0168}\right) = \Phi(-0.6573) = 0.2555$

(c) $\Phi^{-1}(0.95) \times 19.0168 + 362 - 0.5 = 392.7799$

(d) $P(X > 400) = 1 - P(X \leq 400) = 1 - P(X \leq 400 + 0.5) = 1 - \Phi\left(\frac{400.5 - 362}{19.0168}\right) = 1 - \Phi(2.0245) = 0.0215$

Then the probability is $0.0215 = 4.6225 \times 10^{-4}$.

4-73. Let $X$ denote the number of particles in $10 \text{ cm}^2$ of dust. Then, $X$ is a Poisson random variable with $\lambda = 10(1000) = 10,000$. Also, $E(X) = \lambda = 10^3$ and $\text{V}(X) = \lambda = 10^3$.

$P(X > 10,000) \equiv P(Z > \frac{10,000 - 10,000}{\sqrt{10,000}}) = P(Z > 0.005) = 0.498$

4-78. (a) $P(X < 5) = 0.3935$

(b) $P(X < 15 \mid X > 10) = \frac{P(X < 15, X > 10)}{P(X > 10)} = \frac{P(X < 15) - P(X < 10)}{1 - P(X < 10)} = \frac{0.1447}{0.3679} = 0.3933$

(c) They are the same.

4-81. Let $X$ denote the time until the first call. Then, $X$ is exponential and $\lambda = \frac{1}{E(X)} = \frac{1}{15}$ calls/minute.

\[ a) \quad P(X > 30) = \int_{30}^{\infty} e^{-\frac{x}{15}} \, dx = -e^{-\frac{x}{15}}\Big|_{30}^{\infty} = e^{-2} = 0.1353 \]

\[ b) \quad \text{The probability of at least one call in a 10-minute interval equals one minus the probability of zero calls in a 10-minute interval and that is } P(X > 10). \]

\[ P(X > 10) = -e^{-\frac{10}{15}} = e^{-2/3} = 0.5134 \]

Therefore, the answer is 1 - 0.5134 = 0.4866. Alternatively, the requested probability is equal to $P(X < 10) = 0.4866$.

\[ c) \quad P(5 < X < 10) = e^{-2/3} - e^{-1/3} = 0.2031 \]

\[ d) \quad P(X < x) = 0.90 \quad \text{and} \quad P(X < x) = e^{-\frac{x}{15}} \quad \text{so} \quad 1 - e^{-x/15} = 0.90. \quad \text{Therefore, } x = 34.54 \text{ minutes}. \]
4-88. Let $X$ denote the distance between major cracks. Then, $X$ is an exponential random variable with $\lambda = 1/E(X) = 0.2$ cracks/mile.

$$a) \ P(X > 10) = \int_{10}^{\infty} 0.2e^{-0.2x} \ dx = -e^{-0.2x}\bigg|_{10}^{\infty} = e^{-2} = 0.1353$$

b) Let $Y$ denote the number of cracks in 10 miles of highway. Because the distance between cracks is exponential, $Y$ is a Poisson random variable with $\lambda = 10(0.2) = 2$ cracks per 10 miles.

$$P(Y = 2) = \frac{e^{-2}2^2}{2!} = 0.2707$$

c) $\sigma_X = 1/\lambda = 5$ miles.

d) $P(12 < X < 15) = \int_{12}^{15} 0.2e^{-0.2x} \ dx = -e^{-0.2x}\bigg|_{12}^{15} = e^{-2.4} - e^{-3} = 0.0409$

e) $P(X > 5) = -e^{-0.2x}\bigg|_{5}^{\infty} = e^{-1} = 0.3679$. By independence of the intervals in a Poisson process, the answer is $0.3679^2 = 0.1353$. Alternatively, the answer is $P(X > 10) = e^{-2} = 0.1353$. The probability does depend on whether or not the lengths of highway are consecutive.

f) By the memoryless property, this answer is $P(X > 10) = 0.1353$ from part e).

4-95. a) $\Gamma(6) = 5! = 120$

$$b) \ \Gamma\left(\frac{3}{2}\right) = \frac{3}{2} \Gamma\left(\frac{1}{2}\right) = \frac{3}{2} \Gamma\left(\frac{1}{2}\right) = \frac{3}{4} \pi^{1/2} = 1.32934$$

c) $\Gamma\left(\frac{9}{2}\right) = \frac{9}{2} \Gamma\left(\frac{7}{2}\right) = \frac{9}{2} \frac{5}{2} \frac{3}{2} \frac{1}{2} \Gamma\left(\frac{1}{2}\right) = \frac{105}{16} \pi^{1/2} = 11.6317$

4-98. Let $X$ denote the pounds of material to obtain 15 particles. Then, $X$ has an Erlang distribution with $r = 15$ and $\lambda = 0.01$.

$$a) \ E(X) = \frac{r}{\lambda} = \frac{15}{0.01} = 1500 \ \text{pounds.}$$

$$b) \ V(X) = \frac{15}{0.01^2} = 150,000 \ \text{and} \ \sigma_X = \sqrt{150,000} = 387.3 \ \text{pounds.}$$

4-104. $\Gamma(r) = \int_{0}^{\infty} x^{r-1}e^{-x} \ dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$$\Gamma(r) = -x^{r-1}e^{-x}\bigg|_{0}^{\infty} + (r-1)\int_{0}^{\infty} x^{r-2}e^{-x} \ dx = (r-1)\Gamma(r-1).$$